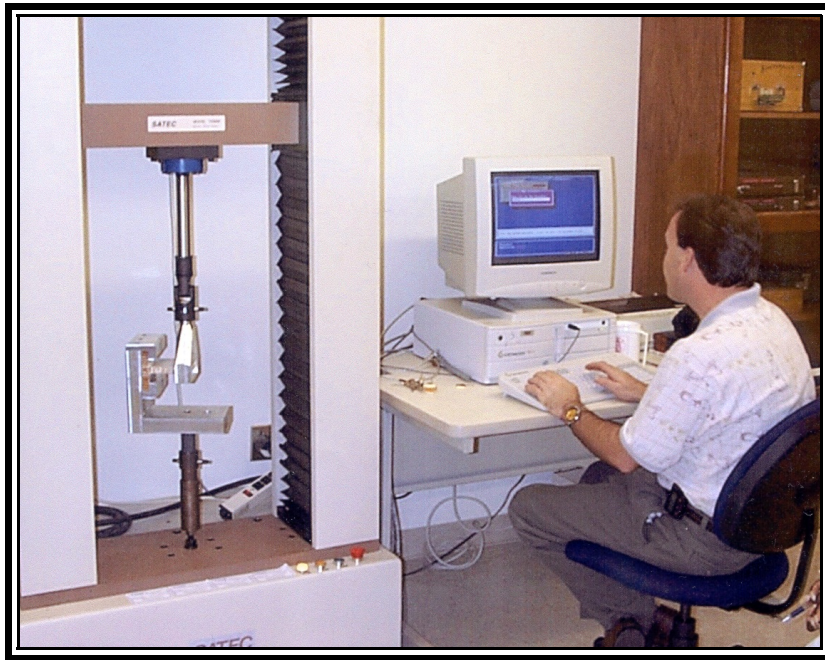


# **MAE/CE 370 - MECHANICS OF MATERIALS - LABORATORY MANUAL**

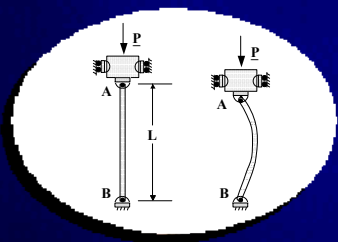
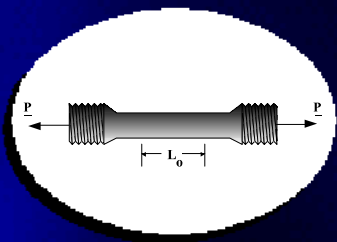


**Version 2.0**

**by**

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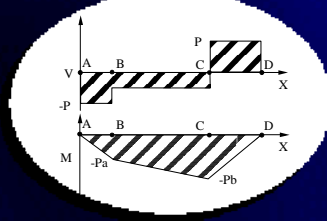
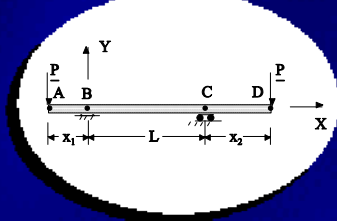
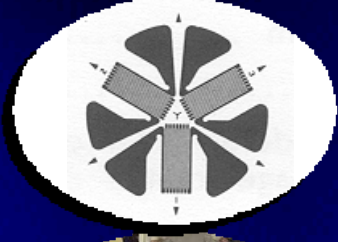
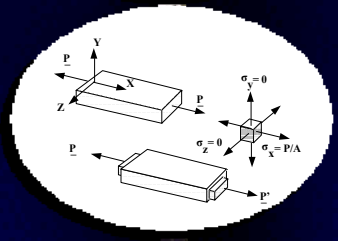
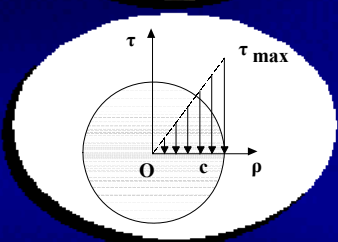
**July 2011**



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MAE/CE 370 Mechanics of Materials Laboratory Manual  
 John A. Gilbert, Ph.D. and Christina L. Carmen, Ph.D.  
 Version 2.0 July 2011



# CHAPTER 1 - INTRODUCTORY REMARKS

## 1.1 Introduction and Copyright

Welcome to the laboratory portion of MAE/CE 370, Mechanics of Materials. Included in this laboratory manual are the instructions for a number of experiments to be performed in conjunction with the course. The description for each experiment includes its objective, an equipment list, background material, and a recommended procedure. Data sheets and calculation sheets have also been prepared for most experiments. Although this relieves the student from preparing these, it does not alleviate the need to include this information in the laboratory reports. Students are highly encouraged to read pertinent sections of this manual before coming to class.

The laboratory experiments, manual, and class notes are continuously being upgraded and constructive criticism is welcome at any time during the semester. Please bring technical problems, and typos or format errors in the notes or manual, to the attention of your instructor so that corrections can be made in later versions.

Portions of this manual were taken directly from technical literature provided by the commercial sources that provided the equipment. **The University of Alabama in Huntsville has been granted license for unlimited copying of the enclosed material but only for distribution to, and use by, the students enrolled in our institution.** Under all other circumstances, no part of this manual may be reproduced in any form or by any means without written permission from the vendors in question.

## 1.2 Laboratory Location and Course Requirements

The Mechanics of Materials Laboratory is located in TH N275. The use of word processing programs, such as Microsoft Word®; and, spread sheets, such as Microsoft Excel®; are highly recommended for preparation of laboratory reports. Students should consider using software such as Microsoft PowerPoint® for making oral reports. The Microsoft® suite is installed in the MAE Computer Laboratories located in TH N204 and TH N278. A login ID is required as well as a password (see <http://www.uah.edu/admin/tag/chargerid.html>); students are responsible for providing their own flash drive. Files may be transmitted elsewhere via ftp or uploaded to sites such as Dropbox®. Please note that temporary directories are not secure and the contents of such partitions may be removed at any time. Important messages for the laboratory instructor during the lab hours can be left in the MAE departmental office: (256) 824-6154.

A laboratory section of MAE/CE 370 must be scheduled to fulfill the requirements of the course. Barring any unforeseen or special circumstances, **students who do not obtain an overall passing grade on the laboratory will fail the class.** Guidelines for preparing and submitting laboratory reports are included in Chapter 2.

The concepts and experiments associated with the laboratory will be incorporated into the examinations given as part of the lecture session. As far as the laboratory session is concerned, all students are required to attend all regularly scheduled laboratory sessions. In case of absence, students are expected to satisfy the laboratory instructor that the absence was for good reason. For excessive cutting of classes, or for dropping the course without following the official procedure, students may fail the course.

### 1.3 Concerns, Complaints, and Evaluations

Issues regarding the laboratory experiments and grading of the laboratory reports should be initially discussed with your laboratory instructor. Problems with the laboratory instructor should be discussed with the faculty responsible for the lecture section and/or the course coordinator. Serious complaints and/or unresolved issues regarding the course and/or the instructors should be brought to the attention of the department chair. Please provide as much documentation as possible when registering a complaint.

Faculty and laboratory instructors are required to distribute student evaluation forms before the end of the semester. Students are highly encouraged to participate in this process and should consider making detailed comments on the back of these forms.

### 1.4 Synopsis of Mechanics of Materials

When loads are applied to a deformable body they produce stresses. The stresses represent the force intensity and are computed by dividing the force by the area over which it acts. A **normal stress** is produced when the force is perpendicular to the surface under consideration. A tensile stress results when the force is directed along the outer normal to the exposed surface; a compressive stress results when the force is directed toward the surface. **Shear stress** results when the force is tangent to the surface.

The stresses produce changes in shape (deformations) characterized by a quantity called strain. Normal stresses produce **normal strains** defined as the change in length of a line segment divided by the original length of the segment. Shear stresses produce **shear strains** defined as the change in angle between two line segments that were originally perpendicular to one another.

There are three basic loading conditions that can be superimposed to produce a general state of deformation. **Axial loading** produces either tensile or compressive stresses assumed to be uniformly distributed across a plane cut normal to the longitudinal axis of the member. Combinations of normal and shear stresses occur on oblique planes. The stresses are computed by dividing the load (P) by the area (A) considered; thus,

$$\sigma = \frac{P}{A} \qquad \tau = \frac{P}{A} \quad . \qquad (1.4-1)$$



**Bending** produces a uniaxial stress condition in which normal stresses occur parallel to the longitudinal axis of the member. For a prismatic member possessing a plane of symmetry, subjected at its ends to equal and opposite couples acting in a plane of symmetry, the stress distribution is linear through the thickness; compressive stresses occur on one side of the neutral axis and tensile stress occur on the other side. The stress is computed using the flexure formula

$$\sigma = \frac{M c}{I} \quad (1.4-2)$$

where M is the moment, c is the distance measured from the neutral axis to the point under consideration, and I is the centroidal moment of inertia measured around the axis about which the moment is applied.

**Torsion** produces shear stresses. In a prismatic member of circular cross section subjected to couples (torques) of magnitude T, the shear stress acts in the direction of the applied torque. It is zero at the center of the shaft and maximizes at the outer surface. The stress is computed using the elastic torsion formula

$$\tau = \frac{T \rho}{J} \quad (1.4-3)$$

where ρ is the distance measured from the center of the shaft to the point under consideration and J is the polar moment of inertia.

Shear stresses also result in prismatic members subjected to transverse loads. In this case, the stress is given by

$$\tau = \frac{V Q}{I t} \quad (1.4-4)$$

where V is the shear force, Q is the first moment of the area measured about the neutral axis of the portion of the cross section located either above or below the point under consideration, I is the centroidal moment of inertia of the entire cross section, and t is the width of a cut made through the point perpendicular to the applied load.

The stress is related to the strain through **constitutive equations** (Hooke's Laws) that depend upon material properties. In general, these are very complex relations especially when dealing with composite materials; however, only two independent material constants [typically Young's modulus (E) and Poisson's ratio (ν)] are required when dealing with a linearly elastic (stress-strain curve is linear), homogeneous (mechanical properties are independent of the point considered), and isotropic (material properties are independent of direction at a given point) material. Other constants, such as the shear modulus [G = E/2(1 + ν)], are sometimes used in these equations. **Material constants** are evaluated under simple loading conditions (tension test, bending test, torsion tests) on specimens having simple geometry (typically rods, bars, and beams).

It is often necessary to find the weakest link in a structure and this can be done by studying the stress distribution at a critical location. This is not as easy as it may sound, since stress is a tensor and must be studied using **transformation equations**. That is, the stress distribution changes from point to point in a loaded member. However, there is usually a critical point or region in the structure where the stress is the highest. The stress distribution at this critical location depends upon the plane considered. There is a set of planes on which the normal stress becomes maximum; shear stress is zero on these planes. The planes of maximum shear typically have a non-zero normal stress component. Analytical equations (transformation equations) or graphical techniques (Mohr's circle) can be employed to pinpoint the maximum stresses and the planes on which they act. These quantities are used in conjunction with failure criteria to assess structural integrity.

In some cases, a member may fail due to its unstable geometry as opposed to the imposed stress state. Column **buckling** is one of the areas where such considerations come into play. Design codes have been developed to prevent these cataclysmic failures.

## **CHAPTER 2 - LAB REPORTS AND GROUP PRESENTATION**

### **2.1 Introduction**

Unless otherwise specified by the instructor, nine (9) laboratory reports, and one (1) group presentation, will be required. Students will be working in groups while performing the experiments but each student is expected to write their own reports.

### **2.2 Laboratory Manual**

A laboratory manual will be provided to each student. These manuals must be returned to the instructor at the end of the semester. Do not record data or write in the manual. The manual contains background material pertaining to each experiment which can still be referenced.

### **2.3 Evaluation of Laboratory Reports**

The primary purpose of an engineering laboratory report is to clearly convey to the reader what was done during the experiment and what conclusions were derived from the results. The report should be concise, well organized, thorough, and neat. A long report is not necessarily the best report. Keep in mind that neatness, grammar, and spelling are taken into account when grading the report.

Nine (9) laboratory reports will be required, each worth 100 points. These reports will be due on the date specified on the lab schedule distributed by your instructor. Late reports will be penalized according to the stipulations set forth by him/her. These penalties will be strictly enforced in fairness to those who submit their reports on time. Late reports must be turned in to the instructors mailbox (TH N273) with an e-mail to the instructor.

### **2.4 Report Guidelines**

The laboratory report must be written in third person. It is encouraged that the report be word-processed. However, you may hand write the report if necessary.

The report must be written in the following format:

- **Title page or first page header:** Include the class title, experiment number, experiment title, date experiment conducted, date report submitted, your name, your group number, and list each members name.

• **Abstract (5 points):** State the purpose of the experiment. Briefly discuss how it was conducted. State the final values obtained and how they compare to theory. If the resulting data is extensive be sure to provide a representative sampling of the final values. Most frequently a percent error is found from:

$$\% \text{ error} = \frac{\text{standard} - \text{measured}}{\text{standard}} \times 100 \quad . \quad (2.4-1)$$

When a standard value is used, be certain to cite the source of that value. Watch significant figures and units.

• **Background (15 points):** The lab instructor will always inform the students at the end of each experiment exactly what to discuss in this section. When asked to discuss a particular equation be sure to discuss the equation (name and purpose) and define each parameter in the equation with the corresponding units.

• **Procedure (15 points):** In this section refer the reader to the lab manual (be sure to list manual in Reference Section) for the exact procedure. However, be sure to list any deviations from the manual's procedure. A sketch of the experimental set-up must be provided and must be neat! Each component must be labeled and the important dimensions should be provided in this section (i.e. anything you measured in the lab as far as lengths, weights, material, etc.).

• **Data and Calculations (35 points):** This section should include the following sub-sections:

RAW DATA: All of the recorded and measured values from the experiment. Be sure to rewrite this information.

CALCULATIONS: Provide all of the required calculated data, plots, tables, etc. These calculations are specified at the end of each laboratory section in the manual. The lab instructor may revise or add to these calculations.

SAMPLE CALCULATIONS: Always provide sample calculations showing how values were obtained and the equations utilized.

• **Results (15 points):** Discuss the final resulting values and how they compare to theoretical values. Are the results as expected? Explain. Discuss at least 5 sources of error which may have affected the data with the entire group before the experiment is over. These sources of error must be listed and explained in this section.

• **Conclusions (5 points):** State whether the resulting values were acceptable (no need to provide specific numbers) and state suggested improvements to the laboratory (i.e. procedure, equipment, etc.). Be sure to provide these and be specific!

- **References (5 points)**

- **Raw Notes (5 points):** Attach the papers that notes were recorded upon during the experiment. There is no need to rewrite these notes.

## 2.5 Presentation Guidelines

The laboratory instructor will break the class up into groups at the beginning of the semester. Each group will be required to make a **15-20 minute long presentation** on a specified experiment. The presentation is worth 100 points and will be given as a review for the class at the end of the semester.

Students are free to use the software of their choice to generate the presentation but are encouraged to use Microsoft PowerPoint®. A video projector and laptop computer will be available. Equipment such as a slide or overhead projector will be provided upon request.

Every person within the group must participate in the exercise. It will be left up to the individual groups to decide the assignment for each member. For example, some members may be responsible for generating the information for the presentation, some may generate the actual slides/presentation, while others may present the material to the class. Remember, each group member will receive the grade that the group receives as a whole.

**Format of Presentation** (Note: The number of slides is only a recommendation.):

Title (1 slide)

Purpose of Experiment

- State the objective. (1 slide)
- Discuss the importance of the experiment. (1 slide)

Background

- Brief discuss the theory. (1 slide)
- Present the pertinent equations. (1 slide) Be sure to define parameters with units.

Equipment (1 or 2 slides)

- Show all equipment, beam dimensions, location of gages, etc.
- No wiring diagrams are necessary.

Data

- Show raw data. (1 slide)
- Describe final results using values, plots, etc. (1 or 2 slides)

Results

- Were the results as expected? Discuss the errors. (1 slide)

Conclusions (1 slide)

- What did you learn from the experiment?
- Any suggestions for improvement?

Group Member Assignments (1 slide)

List the responsibility of each member in preparing for the exercise.



## **2.6 In-Class Examination on Laboratory Concepts**

The concepts and experiments covered in the laboratory will be emphasized during one of the regularly scheduled examinations given as part of the lecture section. This exam will cover topics related to each of the experiments conducted during the laboratory. Unless otherwise stated, the questions will only cover material contained within this manual. The following preparation/study tips are intended to aid the student in preparation for this exam.

The exam will be theoretical with the format consisting of multiple choice questions, short answer questions, short discussion questions, and possible schematic sketches.

### **Preparation/Study Tips:**

- Understand how strain gages function.
- For each experiment be able to visualize the physical set-up and equipment utilized.
- Understand the objective of each experiment.
- Understand the background/theoretical information.
- Recognize and understand the pertinent equations utilized in the calculations.
- Predict and understand trends within any plots generated.
- Provide approximate final values obtained (no need to memorize all resulting numerical values).
- Be familiar with the major sources of error within the experiment.

## CHAPTER 3 - MODULUS OF ELASTICITY TENSION TEST

### 3.1 Objective

The purpose of this experiment is to measure the modulus of elasticity (Young's modulus) of different materials by placing test specimens in uniaxial tension. It should take approximately 20 minutes to test each specimen; a maximum group size of 4 people is recommended.

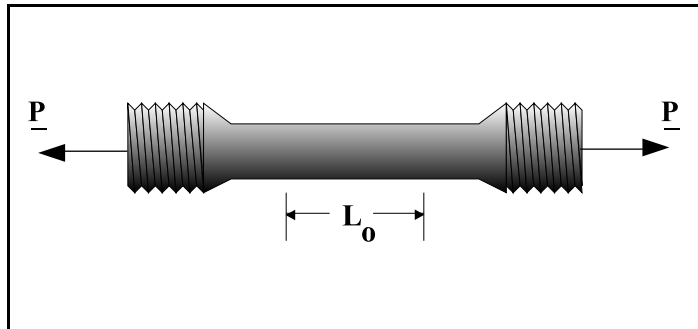


### 3.2 Materials and Equipment

- Tensile testing machine (see Section 12.3 for details)
- Test specimens
- Micrometer (see Section 12.1 for details)
- Calipers (see Section 12.2 for details)

### 3.3 Background

The modulus of elasticity (Young's modulus) is a material constant indicative of a material's stiffness. It is obtained from the stress versus strain plot of a specimen subjected to a uniaxial stress state (tension, compression, or bending). The elastic modulus is used, along with other material constants, in constitutive equations that relate stress to strain in more complex situations.



**Figure 1.** A uniaxial test specimen.

A simple tensile test is the most popular means for determining the elastic modulus. Figure 1, for example, shows a cylindrical test specimen subjected to uniaxial tension. Two reference points, located at a distance  $L_0$  apart, define a gage length. Engineering stress,  $\sigma$ , is computed as the load is increased (based on the original cross sectional area,  $A_0$ ) while engineering strain,  $\epsilon$ , is determined when the elongation experienced by the specimen,  $\delta$ , is divided by the original gage length,  $L_0$ ; and,

$$\sigma = \frac{P}{A_0} \quad \text{and} \quad \epsilon = \frac{\delta}{L_0} . \quad (3.3-1)$$

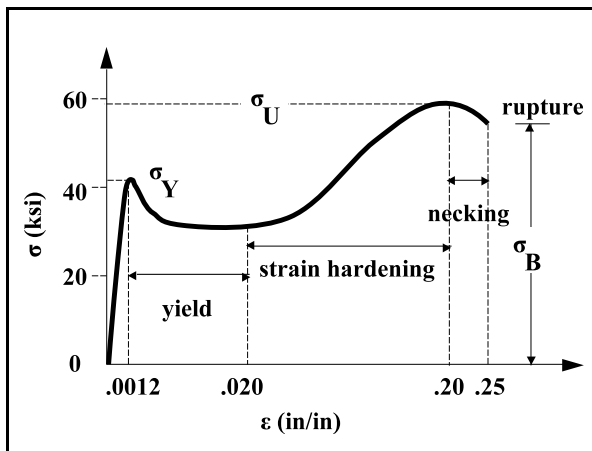
A plot of these quantities produces a stress-strain curve. The modulus of elasticity,  $E$ , is defined as the slope of the linear portion of this curve, and is given by

$$E = \frac{\Delta\sigma}{\Delta\epsilon} \quad (3.3-2)$$

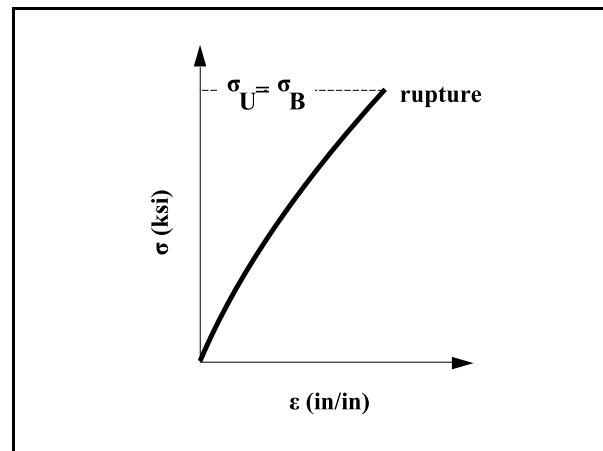
where the stress,  $\sigma$ , is measured in psi (N/m<sup>2</sup> or Pa). In Equation (3.3-2),  $\epsilon$  is the strain measured in in/in (m/m) in the direction of the applied load. Since strain is dimensionless, the elastic modulus is measured in units of psi (Pa).

It is important to realize that Equation (3.3-2) is valid only for uniaxial tension and is a special case of a generalized set of relations known as Hooke's law. Much more complex relations must be used when dealing with more complex loadings.

The shape of the stress-strain curve depends on the material and may change when the specimen is subjected to a temperature change or when the specimen is loaded at a different rate. It is common to classify materials as ductile or brittle. Ductile materials yield at normal temperatures while brittle materials are characterized by the fact that rupture occurs without any noticeable prior change in the rate of elongation. Figures 2 and 3 show typical stress-strain curves for such materials.



**Figure 2.** A stress-strain curve for a ductile material.



**Figure 3.** A stress-strain curve for a brittle material.

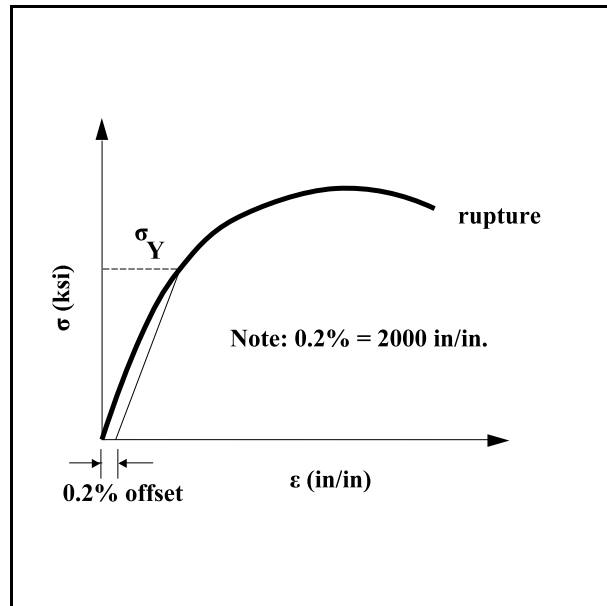
In the case of a ductile material, the specimen experiences elastic deformation, yields, and strain-hardens until maximum load is reached. Necking occurs prior to rupture and failure takes place along the planes of maximum shear stress. Referring to Figure 2, the stress,  $\sigma_y$ , at which yield is initiated is called the *yield stress*. The stress,  $\sigma_u$ , corresponding to the maximum load applied to the specimen is known as the *ultimate strength*. The stress,  $\sigma_B$ , corresponding to rupture is defined

as the *breaking strength*.

In the case of the brittle material characterized by Figure 3, there is no difference between the ultimate strength and the breaking strength. Necking is negligible and failure takes place along the principal planes perpendicular to the maximum normal stress.

Since the slope of the elastic portion of the stress versus strain curve often varies, different methods, such as secant and tangent methods, have been developed to obtain the elastic modulus.

When the yield point is not well defined, a 0.2% offset method is often used to determine the yield stress. As illustrated in Figure 4,  $\sigma_y$  is obtained by drawing a line parallel to the initial straight-line portion of the stress-strain diagram starting from a strain value of  $\epsilon = 0.2\%$  (or  $\epsilon = 0.002$ ). The yield stress is defined as the point where this line intersects the stress versus strain curve.



**Figure 4.** An offset method may be used to determine the yield stress of a material.

The yield stress is defined as the point where this line intersects the stress versus strain curve.

### 3.4 Procedure

The computerized tensile testing machine will be used to produce stress versus strain plots for several different specimens having rectangular cross sections. The data is used to determine the modulus of elasticity while the specimens are examined for failure characteristics.

Information should be entered on the attached work sheet. The steps to be followed are:

1. Measure and record the beam width ( $b$ ), beam thickness ( $t$ ), and length ( $L$ ) of the test section.
2. Mount the specimen in the machine using the grips provided.
3. Boot the computer and select the appropriate test routine to determine the:
  - elastic modulus ( $E$ ) by using the tangent method
  - elastic modulus ( $E$ ) by using the secant method
  - yield stress ( $\sigma_y$ ) by using the 0.2% offset method
  - ultimate strength ( $\sigma_u$ )
  - breaking strength ( $\sigma_b$ )

4. Examine each specimen after it has failed and note the degree of necking and orientation of the fracture surface.

### **3.5 Laboratory Report**

#### **FIGURES:**

Include a figure of the test specimen with the dimensions indicated.

#### **CALCULATIONS:**

1. From your text or another material handbook find the standard value for the modulus of elasticity of the specimens tested. Calculate the percentage error with the value determined by using the tangent method.

#### **DISCUSSION:**

1. What are possible sources of error?
2. Were your errors within reasonable limits ( $< 10\%$ )?
3. Why are the failed specimens shaped as they are?



## WORK SHEET FOR MODULUS OF ELASTICITY TENSION TEST

### BEAM DIMENSIONS:

b = \_\_\_\_\_ inches (width)  
t = \_\_\_\_\_ inches (thickness)  
L = \_\_\_\_\_ inches (length of test section)

### OBSERVATIONS:

Degree of necking: \_\_\_\_\_

Orientation of fracture plane: \_\_\_\_\_

### THEORETICAL COMPARISON:

$$\frac{E_{standard} - E_{experiment}}{E_{standard}} \times 100 = \text{_____ \%}$$

## CHAPTER 4 - COLUMN BUCKLING TEST

### 4.1 Objective

The purpose of this experiment is to verify the Euler buckling equation for steel columns of various lengths subjected to different end conditions. It should take approximately 20 minutes to test each column; a maximum group size of 4 people is recommended.



### 4.2 Materials and Equipment

- Columns of various lengths made from different materials
- Column buckling machine (see Section 12.4 for details)
- Weights
- Dial indicators

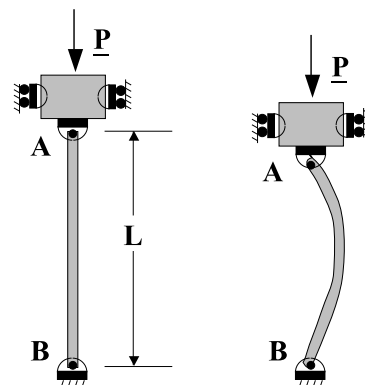
### 4.3 Background

There are usually two primary concerns when analyzing and designing structures: (1) the ability of the structure to support a specified load without experiencing excessive stress and (2) the ability of the structure to support a given load without undergoing unacceptable deformation. In some cases, however, stability considerations are important especially when the potential exists for the structure to experience a sudden radical change in its configuration. These considerations are typically made when dealing with vertical prismatic members supporting axial loads. Such structures are called columns.

A column will buckle when it is subjected to a load greater than the critical load denoted by  $P_{cr}$ . That is, instead of remaining straight, it will suddenly become sharply curved as illustrated in Figure 1.

The critical load is given in terms of an effective length by,

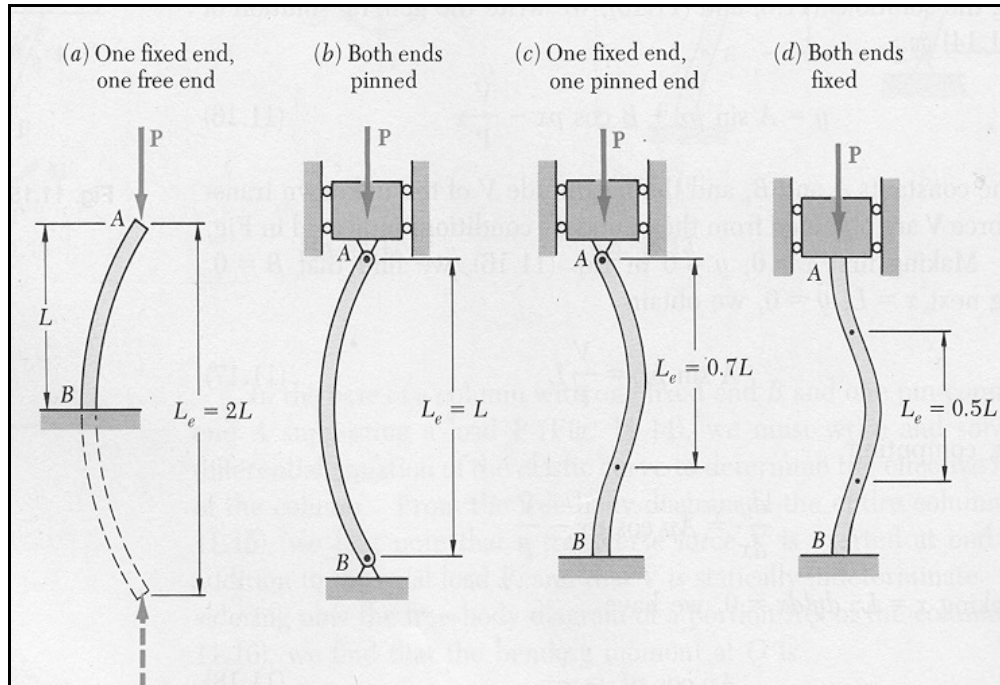
$$P_{cr} = \frac{\pi^2 E I}{L_e^2} \quad (4.3-1)$$



**Figure 1.** A column will buckle when a critical load is reached.

where  $E$  is the elastic modulus,  $I$  is the moment of inertia, and  $L_e$  is the effective length.

The expression in Equation (4.3-1) is known as Euler's formula. The effective length depends upon the constraints imposed on the ends of the column. Figure 2 shows how the effective length is related to the actual length of the column for various end conditions.



**Figure 2.** Effective lengths of columns for various end conditions; figure taken from Beer, Johnson, DeWolf, and Mazurek, "Mechanics of Materials."

The critical load is computed by making  $I = I_{\min}$  in Equation (4.3-1). Thus, if buckling occurs, it will take place in a plane perpendicular to the corresponding principal axis of inertia.

The radius of gyration,  $r$ , is often introduced into Euler's formula. This quantity is given by

$$r = \sqrt{\frac{I_{\min}}{A}} \quad (4.3-2)$$

where  $A$  is the cross sectional area of the column.

Substituting Equation (4.3-2) into (4.3-1),

$$P_{cr} = \frac{\pi^2 E A}{(L_e/r)^2} \quad (4.3-3)$$

The value of the stress corresponding to the critical load is called the critical stress. This quantity is given by

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/r)^2} . \quad (4.3-4)$$

In Equations (4.3-3) and (4.3-4), the quantity  $(L_e / r)$  is called the slenderness ratio of the column.

For long columns, with a large slenderness ratio, Euler's formula is adequate for design purposes. However, for intermediate and short columns, where failure occurs essentially as a result of yield, empirical formulas are used to approximate test data. These empirical formulas are specified on the basis of material tests conducted by engineers working in that field. The American Institute of Steel Construction, for example, sets the design standards for structural steel in the United States.

#### 4.4 Procedure

The number of columns tested is determined by the instructor. Ideally, several lengths with three different end conditions [see Figure 2, cases (b), (c), and (d)] would be tested, but normally this is not feasible.

The most critical factor in this lab is to ensure that the columns are loaded in a perfectly vertical fashion. Any angular rotation (especially in the case when both ends are fixed) will result in erroneous results. Care should also be taken in adjusting the collar on the post for each column. Do not drop the loading beam onto the column, as it may damage it enough to affect the results. It is important to stop the loading of the column as soon as the critical load has been reached to avoid permanent damage to the column.

For each column tested:

1. Measure and record the dimensions of the column on the worksheet included on page 4.6.
2. Calculate the expected buckling load for the end conditions at hand. The steps for doing this are outlined on the worksheet.
3. Orient the satin chrome blocks on the loading frame for the end conditions chosen. V-notches should face away from the mounting surface (towards the column) for pinned ends and towards the mounting surface (away from the column) for fixed ends.
4. With the end conditions selected, adjust the capstan nut (with four hand knobs) located on the left vertical post of the loading frame until the upper surface of the nut is flush with the threaded sleeve. This is to provide adequate adjustment to level the loading beam at each

applied increment of load, and must be rechecked each time the column is changed.

5. The loading beam should then be adjusted to the desired column as follows:
  - The stop for the loading beam (sleeve with rubber bumper on right hand vertical post) should be adjusted to approximately two inches below the length of the column selected. This stop is only used to rest the loading beam when changing columns and should be adjusted so that it will not interfere with the test.
  - Lower the entire loading beam onto the column by loosening the split nut on the left post (below the spring) and slide the collar on the post to position the beam for the column selected. This should be accomplished using both hands, since the beam assembly is not light and, if allowed to strike the column, may damage the column sufficiently to affect the test.
  - Once the column is contacted, the loading beam should be approximately leveled (refer to spirit level on the beam) before the split lock is tightened. The leveling adjustment may be accomplished by use of the capstan nut. Be sure the split lock is tightened; if not, the pivot end of the loading beam will slide up the post during the test and erroneous readings will result.
6. Attach the appropriate connecting link between the load scale and the loading handwheel. The weight of the loading beam must be counter-balanced by using weights and the pulley at the top of the right-hand vertical post. Adjust the balance weights until the connecting link between the load scale and the loading handwheel just rises to contact the shackle at the top of the loading handwheel. Zero the loading scale with the aluminum knob protruding from its top. **It should be noted that the difference in the moment arms of the applied load and the load transmitted to the column has been taken into account in the calibration of the load scale.** The force read from the load scale is the force applied to the column.
7. After the column is in position, the dial indicator is installed in the brackets and fastened to the center post. The indicator bracket should be moved up or down the post so that the indicator point contacts the column at its midpoint. The indicator may then be zeroed by loosening the black plastic knob that holds the indicator on the frame and then moving it gently toward the column until the needle on the small scale is zero. The large scale is zeroed by rotating the outside bezel until the large needle is on zero. One revolution on the large scale is 0.100 in. (2.54 mm) and is equal to 1 on the small scale. Each graduation of the large scale is 0.001 in. (0.025 mm). Extreme care should be exercised in handling the dial indicator.
8. In order to assure that the column will deflect away from the dial indicator, brass weights are utilized to apply a small side load to the column. The pulley on the left post should be adjusted so that the tension wire attached to the clip on the column is horizontal. The clip



should be positioned on the column so that the notch in the back side of the clip surrounds the contact point of the dial gage. Apply a 0.3 lb (1.34 N) horizontal load in the center of 21 in. (53 cm), 24 in. (61 cm), and 30 in. (76 mm) long columns, and 0.7 lb (3.12 N) to the 15 in. (38 mm) and 18 in. (46 mm) long columns.

9. Apply the load to the column by means of the handwheel which operates the loading screw. After each increment of load, record the load and deflection on the data sheet. Suitable increments for the loading of the column may be obtained by rotating the handwheel so that the spirit level on the loading beam shifts approximately 0.5 in. (12.7 mm). The beam can then be leveled with the capstan nut. (In order to achieve more data points, particularly at higher loads, the handwheel should be rotated less.)
10. When two consecutive readings of the load on the load scale differ by less than 2 lb (8.9 N), the column may be considered to have buckled and the critical load should be recorded.

#### **4.5 Laboratory Report**

##### **FIGURES:**

Include sketches of the columns with dimensions and end conditions indicated.

##### **CALCULATIONS:**

1. Plot the load versus deflection for each test.
2. Compare the observed buckling load (load value where the load versus deflection plot levels out) to the calculated theoretical buckling load computed from Equation (4.3-3).
3. Discuss possible sources of error.

## WORK SHEET FOR LOADING OF COLUMNS

COLUMN DIMENSIONS:

L = \_\_\_\_\_ inches (actual length)  
 b = \_\_\_\_\_ inches (width)  
 t = \_\_\_\_\_ inches (thickness)  
 A = \_\_\_\_\_ square inches (area = b t)

EFFECTIVE LENGTH:

$L_e$  = \_\_\_\_\_ inches (effective length); *i.e.*, modify L by taking end conditions into account.

MINIMUM MOMENT OF INERTIA:

$$I_{\min} = \frac{b t^3}{12} = \text{_____ } in^4$$

RADIUS OF GYRATION:

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{(\text{_____})}{(\text{_____})}} = \text{_____ } in.$$

SLENDERNESS RATIO:

$$\frac{L_e}{r} = \frac{(\text{_____})}{(\text{_____})} = \text{_____}$$

ESTIMATE OF CRITICAL LOAD (assuming  $E = 30 \times 10^6$  psi):

$$P_{cr} = \frac{\pi^2 E A}{(L_e/r)^2} = \frac{(3.1416)^2 (30 \times 10^6) (\text{_____})}{(\text{_____})^2} = \text{_____ } lb$$

COMPARISONS:

$$\frac{P_{cr \text{ theory}} - P_{cr \text{ experiment}}}{P_{cr \text{ theory}}} \times 100 = \text{_____ } \%$$

TEST DATA:

Deflection (in.)	Load (lb)

## CHAPTER 5 - TORSION TEST

### 5.1 Objective

The purpose of this experiment is to test torsion specimens, made of different materials, to failure. The objective of each test is to determine the modulus of rigidity, establish the shear stress at the limit of proportionality, and verify the analytical relation between the applied torque and the angle of twist. It should take approximately 15 minutes to test each specimen; a maximum group size of 4 people is recommended.

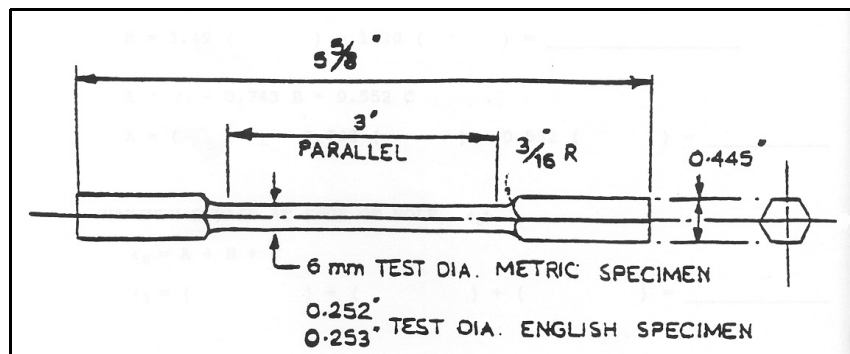


### 5.2 Materials and Equipment

- Torsion testing machine (see Section 12.5 for details)
- Torsion test specimens (aluminum, brass, steel, etc.)
- Micrometer (see Section 12.1 for details)
- Calipers (see Section 12.2 for details)

### 5.3 Background

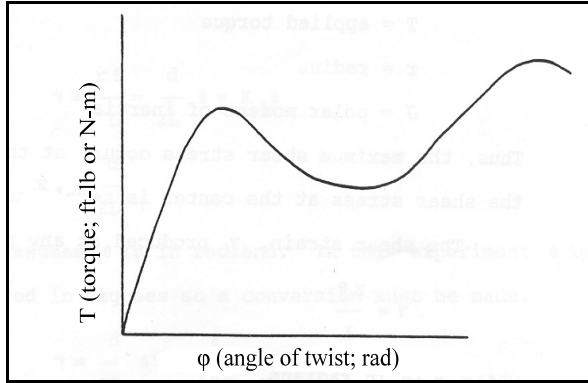
This experiment relies on a bench mount torsion testing machine that utilizes standard 0.2525 in. (6 mm) diameter cylindrical test specimens. As illustrated in Figure 1, each specimen is equipped with hexagonal ends that fit into precision chucks designed to hold the hexagonal section.



**Figure 1.** A typical torsion specimen.

The modulus of rigidity, or shear modulus,  $G$ , may be obtained from a torque-twist curve, such as that illustrated in Figure 2. The curve is similar in shape and character to the stress-strain curve obtained from an axial test, but it is drawn from torsion test data.

A torsion test consists of taking torque and relative twist values over a test length of a specimen with



**Figure 2.** Typical results from a torsion test.

a circular cross section. The yield stress, proportional limit, elastic limit and constant of proportionality in the elastic region are found in a similar manner to the properties from an axial test, but the torsion formulae must be used.

Referring to Figure 2, the shear modulus can be computed using

$$G = \frac{T L}{\phi J} \quad (5.3-1)$$

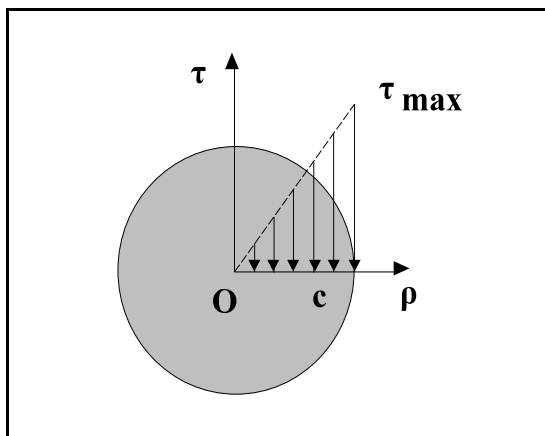
where the quantity  $T/\phi$  is the slope of the linear portion of the curve in the region below the proportional limit. In Equation (5.3-1),  $T$  is the torque,  $\phi$  is the angle of twist measured in radians,  $L$  is the length of the specimen, and  $J$  is the polar moment of inertia. For a solid shaft of circular cross section

$$J = \frac{\pi}{2} R^4 = \frac{\pi d^4}{32} \quad (5.3-2)$$

where  $R$  is the radius, and  $d$  is the diameter of the shaft. The terms in Equation (5.3-1) can be rearranged to show that

$$\phi = \frac{T L}{J G} \quad (5.3-3)$$

The shear stress,  $\tau$ , at any point in the shaft is in a direction consistent with the direction of the applied torque and is given by the elastic torsion formula as,



**Figure 3.** Shear stresses in a circular shaft.

$$\tau = \frac{Tr}{J} \quad (5.3-4)$$

where  $r$  is the radial distance measured outward from the center of the shaft.

It is evident from Equation (5.3-3) that shear stress increases linearly with distance. As illustrated in Figure 3, the maximum shear stress occurs at the outer surface of the shaft and decreases to zero at its center. In this case, a clockwise torque was applied to the shaft.



The shear stress produces a shear strain that can be expressed as

$$\gamma = \frac{r \theta}{L} \quad (5.3-5)$$

where  $\theta$  is measured in radians.

The shear modulus can also be obtained directly from measurements made from a graph of shear stress versus shear strain. Since  $G$  is defined as the slope of the linear region,

$$G = \frac{\tau}{\gamma} \quad \text{or} \quad \tau = G \gamma \quad (5.3-6)$$

A simplified expression for the maximum shear stress on the surface of a solid circular specimen can be obtained by combining Equations (5.3-2) and (5.3-4). Since  $r = d/2$  there,

$$\tau = \frac{Tr}{J} = \frac{T \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16 T}{\pi d^3} = K_1 T \quad (5.3-7)$$

where the constant  $K_1$  is given by

$$K_1 = \frac{16}{\pi d^3} \quad (5.3-8)$$

An expression for the shear strain at the surface can be found by letting  $r = d/2$  in Equation (5.3-5) as

$$\gamma = \frac{r \phi}{L} = \frac{\frac{d}{2} \phi}{L} = \frac{d}{2 L} \phi = K_2 \phi \quad (5.3-9)$$

where the constant  $K_2$  is given by

$$K_2 = \frac{d}{2 L} \quad (5.3-10)$$

The expression in Equation (5.3-9) assumes that  $\phi$  is measured in radians. Since this quantity is

measured in degrees while conducting the experiment, a conversion must be made. With  $\theta$  in degrees, Equations (5.3-9) and (5.3-10) become

$$\gamma = \frac{d}{2L} \phi \frac{\pi}{180} = K'_2 \phi \quad (5.3-11)$$

and

$$K'_2 = \frac{d}{2L} \frac{\pi}{180} \quad (5.3-12)$$

## 5.4 Procedure

1. Measure and record the test length and test diameter of the specimen. Also record the material type.
2. Draw a line down the length of the test specimen using a pencil. This will enable the degree of twist to be observed during the test.
3. Mount the specimen firmly in the torsion testing machine by following these steps:
  - Allow the spring balance to hang free of the torque arm and zero the balance by adjusting the small knurled screw at the top right hand of the balance.
  - Slide the balance along the supporting framework until the two engraved lines, one on the horizontal cross member of the frame and the other on the balance assembly block (which is sliding along this member) coincide.
  - Slide the hook of the balance under the knife edge on the torque arm with the hook hanging free at its lowest position.
  - Clamp the specimen into the jaws of the torsion machine. It is essential that the whole length of the hexagon ends of the specimen is contained fully within the chuck jaws.
  - When the specimen has been firmly fixed in position, clamp the straining head to the bed.
  - Turn the handle on the straining head until the torque arm is in the horizontal as shown by the spirit level.
  - Turn the spring balance hand wheel to raise the balance until the hook on the balance is just

contacting the knife edge on the torque arm. This will be seen by movement of the spirit level bubble. Care should be exercised to carry out this operation so that the torque arm and spring balance are "zeroed" -- both the balance hand wheel and straining head hand wheel may have to be adjusted together to obtain this condition.

- Zero the fine and coarse angular displacement dials on the input and output shafts of the straining head. A knurled nut is provided behind each dial to lock the dial in position. If these cannot be zeroed, record the initial values of both the fine and coarse dial.
  - Zero the revolution counter by turning clockwise. The apparatus is now ready for use and the test specimen can be loaded.
4. The test is performed by rotating one end of the specimen through progressively higher angular displacements. After each angular displacement is applied balance the spirit level using the spring balance hand wheel. After balancing the wheel, record the torque,  $T$  (in-lb), and the angle of twist,  $\phi$  (degrees). Note: Use  $\frac{1}{4}^\circ$  increments for the first  $8^\circ$  of twist and then  $2^\circ$  increments up to  $30^\circ$ .
  5. After applying  $30^\circ$  of rotation, it is no longer necessary to pause and balance the spirit level with the spring balance hand wheel. Turn the straining head handwheel at a consistent pace. At angular increments of approximately  $90^\circ$ , obtain and record the torque along with the corresponding angle of twist. Continue this procedure until failure.

## 5.5 Laboratory Report

### FIGURES:

Include a sketch of the torsion specimen with dimensions indicated.

### CALCULATIONS:

1. Transform torque-angle data to shear stress-shear strain data.
2. Plot the shear stress ( $\tau$ ) versus the shear strain ( $\gamma$ ).
3. Determine the shear modulus ( $G$ ) using your graph.
4. From your graph determine the shear stress at the limit of proportionality,  $\tau_{\text{prop}}$ .
5. Compare  $G$  and  $\tau_{\text{prop}}$  determined experimentally with a standard value.

## WORK SHEET FOR TORSION TEST

BEAM DIMENSIONS:

L = \_\_\_\_\_ inches (test length)

d = \_\_\_\_\_ inches (specimen diameter)

MATERIAL: \_\_\_\_\_

TEST DATA:

Angle of twist, $\phi$ (degrees)	Applied torque, T (in-lb)

## CALCULATION SHEET FOR TORSION TEST

### 1. TRANSFORMATION OF DATA:

MATERIAL: \_\_\_\_\_

$$K_1 = \frac{16}{\pi d^3} = \text{_____ } in^{-3} \quad K'_2 = \frac{d}{2 L} \frac{\pi}{180} = \text{_____}$$

DATA POINTS FOR  $\tau$  VERSUS  $\gamma$  :

Shear Stress (psi) - [ $\tau = K_1 T$ ]	Shear Strain (in./in.) - [ $\gamma = K'_2 \theta$ ; $\theta$ in deg.]

2. PLOT SHEAR STRESS VERSUS SHEAR STRAIN:

Include a plot here.

3. SHEAR MODULUS:

$$G = \frac{\Delta \tau}{\Delta \gamma} = \frac{(\quad)}{(\quad)} = \text{_____ } psi$$

4. LIMIT OF PROPORTIONALITY:

$$\tau_{prop} = \text{_____ } psi$$

5. COMPARISONS:

$$\frac{G_{standard} - G_{experiment}}{G_{standard}} \times 100 = \text{_____ } \%$$

$$\frac{\tau_{standard} - \tau_{experiment}}{\tau_{standard}} \times 100 = \text{_____ } \%$$

Source for standard value: \_\_\_\_\_ .

## CHAPTER 6 - MODULUS OF ELASTICITY FLEXURE TEST

### 6.1 Objective

The purpose of this experiment is to measure the modulus of elasticity (Young's modulus) of an aluminum beam by loading the beam in cantilever bending. The experiment should take approximately 30 minutes to run; a maximum group size of 4 people is recommended.



### 6.2 Materials and Equipment

- Cantilever flexure frame (see Section 12.6 for details)
- No. B101 (2024-T6 high-strength aluminum alloy beam); 1/8 x 1 x 12.5 in. (3x25x320 mm)
- P-3500 strain indicator or equivalent (see Section 12.9 for details)
- Micrometer (see Section 12.1 for details)
- Calipers (see Section 12.2 for details)
- Scale
- Weights and hanger

### 6.3 Background

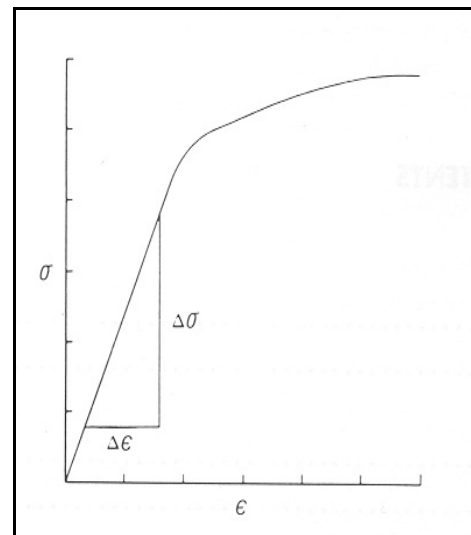
The modulus of elasticity, or Young's modulus, is a material constant indicative of the material's stiffness. It is obtained from the stress versus strain plot of a specimen subjected to a uniaxial stress state (tension, compression, or bending).

Figure 1, for example, shows a typical "stress-strain" diagram for a metal under uniaxial stress. For materials such as aluminum, strain is an essentially linear function of the stress up to the point at which the material yields.

The modulus of elasticity,  $E$ , is defined as the slope of the linear portion of the diagram, and is given by

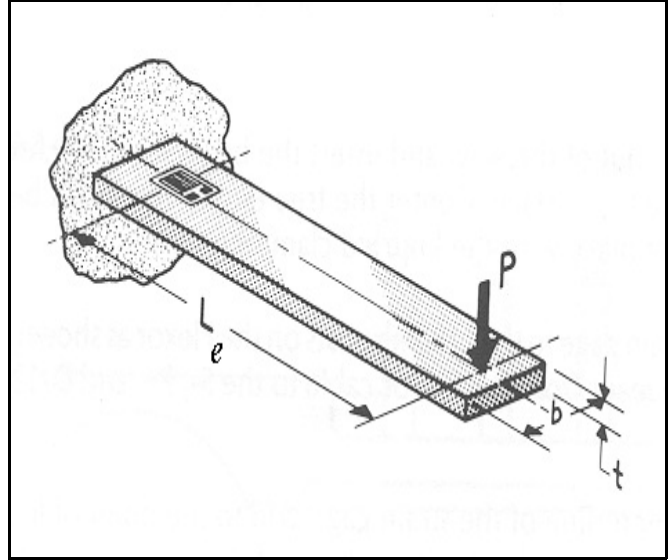
$$E = \frac{\Delta\sigma}{\Delta\epsilon} \quad (6.3-1)$$

where  $\sigma$  is the stress measured in psi ( $\text{N/m}^2$  or Pa). In Equation (6.3-1),  $\epsilon$  is the strain measured in in/in (m/m). Thus, the elastic modulus is measured in units of psi (Pa).



**Figure 1.** A stress versus strain curve for a ductile material.

A uniaxial stress state is obtained on the surface of a cantilever beam when it is loaded at its free end. The loading condition, illustrated in Figure 2, places the beam in a combined shear and bending state but the shear stresses are zero on the upper and lower surfaces. The bending stresses are directed along the longitudinal axis of the beam; they maximize on the upper surface and decrease linearly through the thickness. When the beam has a rectangular cross-section, the magnitude of the tensile stress on the upper surface is equal to that of the compressive stress on the lower surface.



**Figure 2.** An end loaded cantilever beam.

The strain corresponding to the uniaxial bending stress on the free surface of the beam can be measured by placing a strain gage there. In general, the normal bending stress on the surface of the beam,  $\sigma$ , can be calculated by using the flexure formula

$$\sigma = \frac{M c}{I} \quad (6.3-2)$$

where  $M$  is the bending moment at the point of interest measured in in-lb (N-m). In Equation (6.3-2),  $c$  is the distance from the neutral axis to the surface measured in inches (m), and  $I$  is the centroidal moment of inertia measured in in<sup>4</sup> (m<sup>4</sup>) around a horizontal axis.

The neutral axis of the beam shown in Figure 2 is located at the geometric center of the beam. Therefore “ $c$ ” is simply half the beam thickness. The centroidal moment of inertia depends upon the geometry and is given by  $I = bt^3/12$ . The bending moment varies over the length of the beam; it is maximum at the support and zero under the load. The detailed analysis, included in Chapter 8, reveals that the moment at the location of the gage is equal to the load,  $P$ , multiplied by the effective length,  $L_e$ . The latter is defined in Figure 2 as the distance between the gage and the point at which the load is applied.

Substitution of these values into Equation (6.3-2) yields

$$\sigma = \frac{M c}{I} = \frac{PL \frac{t}{2}}{\frac{bt^3}{12}} = \frac{6PL_e}{bt^2} \quad (6.3-3)$$

It must be emphasized that Equation (6.3-3) is only valid on the surface of an end loaded cantilever

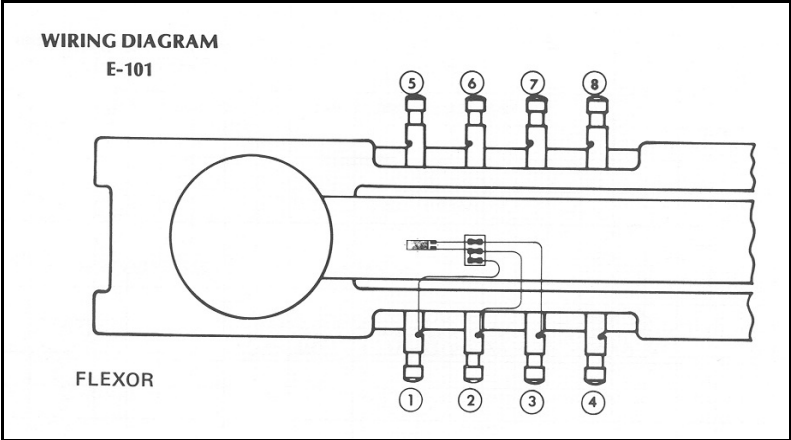


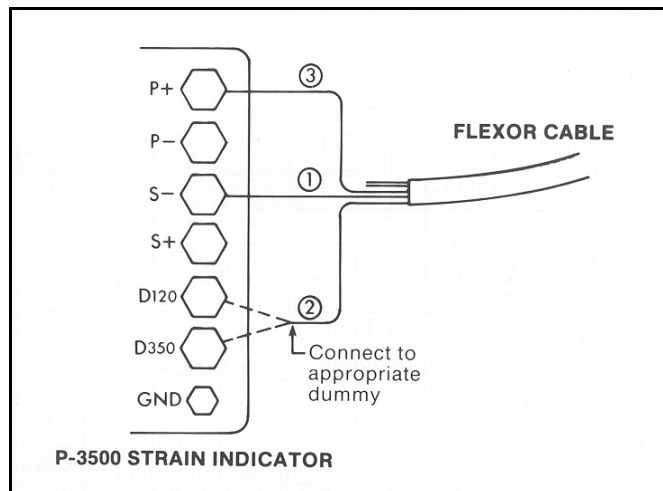
beam with a rectangular cross-section.

## 6.4 Procedure

The surface strain at the section of interest will be measured by a strain gage bonded at that point. The load will be applied in increments, and the corresponding strains recorded. The stresses [calculated from Equation (6.3-3)] and strains will be plotted to produce a stress-strain diagram from which the modulus of elasticity is determined.

Information should be entered on the attached work sheet. The steps to be followed are:

1. Measure and record the beam width ( $b$ ), beam thickness ( $t$ ), and effective length ( $L_e$ ).
  2. Record the gage factor,  $S_g$ , indicated on the beam.
- 
- Figure 3.** Wiring diagram for Experiment No. 1.
3. Using Equation (6.3-3), determine the load,  $P$ , to be applied for a stress,  $\sigma$ , of 15,000 psi to result at the strain gage. This is the maximum load that can be safely applied to the beam without exceeding the yield stress, and is defined as  $P_{max}$  (a few pounds).
  4. With the gaged end of the beam near the support, center the beam in the flexure frame and firmly clamp the beam in place.
  5. Referring to Figure 3, connect the lead wires from the strain gage to the posts on the sides of the "flexor" frame. Referring to Figure 4, connect the appropriate gage leads from the Flexor cable to the S-, P+, and D-120 binding posts of the P-3500 strain indicator. Note: The strain gage employed in this experiment is used in a "quarter-bridge" arrangement by connecting the lead labeled as 2 to the D120 post on the P-3500.



**Figure 4.** Connections to the P-3500.

6. Using Section 12.9 for guidance, depress the Amp Zero button and balance the amplifier. Then depress the Gage Factor button and set the gage factor (as displayed in the LCD readout) to the value given on the strain gage package data form. Select the X1 MULT position and depress the RUN push button. With the beam unloaded (except by its own weight and the weight of the loading hook), use the balance controls of the P-3500 to achieve a bridge balance (as indicated by a zero in the LCD readout). Do not adjust the balance controls again for the remainder of the experiment.
7. Determine ten distinct loads to be applied to the beam, not exceeding  $P_{\max}$  determined in step 3. These need not be equal increments but are determined from the available weights and  $P_{\max}$ .
8. Apply the calibrated load in 10 steps, or increments. At each increment, record the indicated strain and corresponding load on the worksheet. Unload the beam in 10 decrements and again record the load and strain at each decrement.

#### **4.5 Laboratory Report**

##### **FIGURES:**

Include a figure of the beam with the dimensions indicated and the location and orientation of the strain gage clearly shown.

##### **CALCULATIONS:**

1. Calculate the beam stress at the gage for the applied loads using the flexure formula.
2. Make a stress vs. strain graph by plotting the (stress, strain) points. Use both the increasing and decreasing load data.
3. From your graph determine the modulus of elasticity. Be certain to watch units.
4. From your text or another material handbook find the standard value for the modulus of elasticity of 2024-T6 aluminum. Calculate the percentage error in your experiment.

##### **DISCUSSION:**

1. What are possible sources of error?
2. Were your errors within reasonable limits ( $< 10\%$ )?
3. Were your increasing and decreasing graphs the same? Why not?

## WORK SHEET FOR MODULUS OF ELASTICITY FLEXURE TEST

BEAM DIMENSIONS:

b = \_\_\_\_\_ inches (width)

t = \_\_\_\_\_ inches (thickness)

$L_e$  = \_\_\_\_\_ inches (effective length; from gage centerline to applied load)

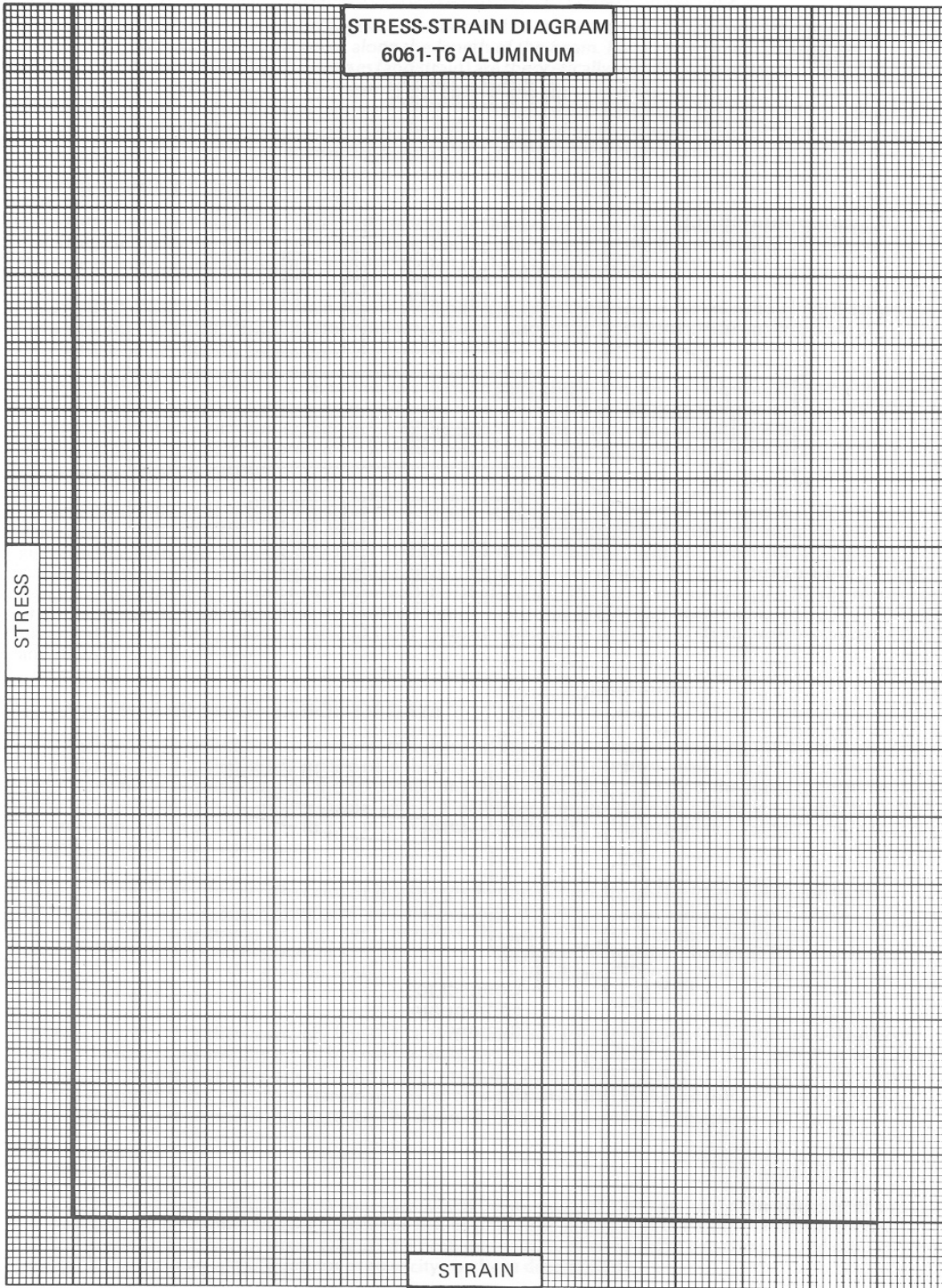
GAGE FACTOR ( $S_g$ ): \_\_\_\_\_

MAXIMUM LOAD (for 15,000 psi):

$$P_{\max} = \frac{\sigma b t^3}{6 L_e} = \frac{(15,000 \text{ lb/in}^2) ( \quad \text{in.} ) ( \quad \text{in.} )^3}{6 ( \quad \text{in.} )} = \quad \text{lb}$$

TABULATION OF LOADS, STRAINS, AND STRESSES:

LOAD (lb)	STRAIN ( $\mu\epsilon$ ) (INCREASING LOAD)	STRESS (psi) (INCREASING LOAD)	STRAIN ( $\mu\epsilon$ ) (DECREASING LOAD)	STRESS (psi) (DECREASING LOAD)
0	0	0		



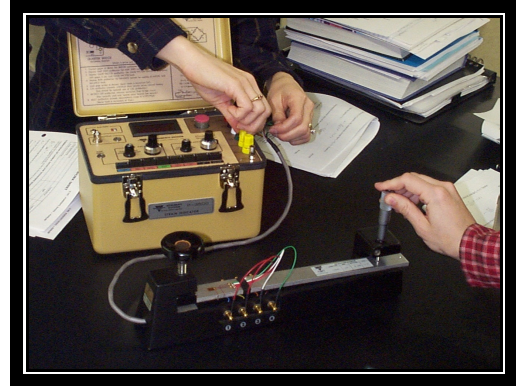
## CHAPTER 7 - POISSON'S RATIO FLEXURE TEST

### 7.1 Objective

The purpose of this experiment is to measure the Poisson's Ratio of an aluminum beam by loading it as a cantilever. The experiment should take approximately 20 minutes to run; a maximum group size of 4 people is recommended.

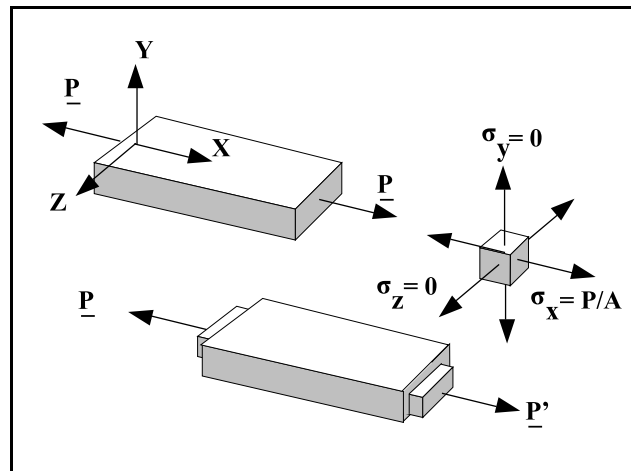
### 7.2 Materials and Equipment

- Cantilever flexure frame (see Section 12.6 for details)
- No. B102 (2024-T6 high-strength aluminum alloy beam); 1/4 x 1 x 12.5 in. (6x25 x320 mm)
- P-3500 strain indicator (see Section 12.9 for details)



### 7.3 Background

When a linearly elastic (stress-strain curve is linear), homogeneous (mechanical properties are independent of the point considered), and isotropic (material properties are independent of direction at a given point) material is subjected to uniaxial stress, the specimen not only deforms in the direction of the applied load, but also exhibits deformation of the opposite sign in the perpendicular direction. In Figure 1, for example, the stress along the x direction *produces an elongation* and a longitudinal axial strain in the direction of the force. The normal stresses on faces perpendicular to the y and z directions are zero; however, *contractions* occur along these transverse directions giving rise to lateral strains.



**Figure 1.** The applied load causes the bar to elongate in the direction of the load and to contract in the transverse directions.

This phenomena, referred to as the Poisson's effect, is characterized by a material property called the Poisson's ratio. The Poisson's ratio is a dimensionless quantity defined as,

$$\nu = - \left[ \frac{\epsilon_{lateral}}{\epsilon_{longitudinal}} \right] \quad (7.3-1)$$

but only for test specimens that are in the uniaxial stress condition. The lateral and longitudinal strains in Equation (7.3-1) must be measured at the same location, in directions perpendicular and parallel to the applied load, respectively. Referring to Figure 1,

$$\nu = - \frac{\epsilon_y}{\epsilon_x} = - \frac{\epsilon_z}{\epsilon_x} . \quad (7.3-2)$$

The **uniaxial stress** condition was discussed in Chapter 3 where it was demonstrated that stress was related to strain through the Young's modulus, E; such that,

$$\sigma_x = E \epsilon_x . \quad (7.3-3)$$

This constitutive equation (often referred to as Hooke's Law), and Equations (7.3-1) and (7.3-2), only hold for this relatively simple stress state. Rearranging Equation (7.3-3) and introducing Equation (7.3-2),

$$\epsilon_x = \frac{\sigma_x}{E} \quad \epsilon_y = \epsilon_z = - \frac{\nu \sigma_x}{E} . \quad (7.3-4)$$

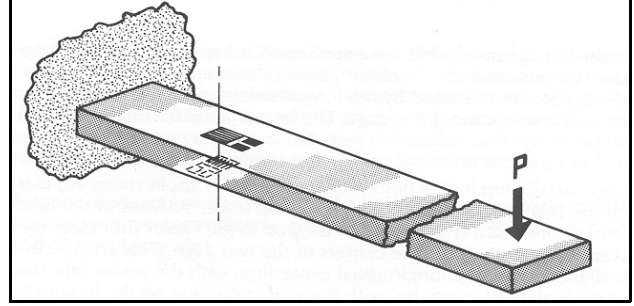
Just like Young's modulus, Poisson's ratio is a constant when dealing with a linearly elastic, homogeneous, and isotropic material. Although E and  $\nu$  are determined by applying relatively simple loading conditions to specimens of simple geometry, these material properties are used in the constitutive equations developed for specimens subjected to more complex loadings. In the case of **biaxial (plane) stress** in the x-y plane, for example, Hooke's Law may be written as,

$$\sigma_x = \frac{E}{1 - \nu^2} ( \epsilon_x + \nu \epsilon_y ) \quad \sigma_y = \frac{E}{1 - \nu^2} ( \epsilon_y + \nu \epsilon_x ) . \quad (7.3-5)$$

Again, it is important to realize that Equations (7.3-1) through (7.3-4) do not hold true for biaxial loading, and other cases different from uniaxial loading. Furthermore, the form of the relations for plane stress presented in Equation (7.3-5) incorporates the knowledge that  $\sigma_z = 0$ , and may be misleading, since the strain component normal to the surface,  $\epsilon_z$ , does not appear there. In general, this strain component is not equal to zero. When  $\sigma_z = \tau_{xy} = \tau_{xz} = 0$  (plane stress),

$$\epsilon_z = - \frac{\nu}{(1 - \nu)} [\epsilon_x + \epsilon_y] . \quad (7.3-6)$$

As illustrated in Figure 2, the experiment in question relies on a cantilever beam in bending with one strain gage mounted on the top surface (longitudinally) and the other mounted on the bottom surface (laterally). The fibers on the upper and lower surfaces of the beam are subjected to a uniaxial stress state consisting of normal stresses that occur parallel to the longitudinal axis of the beam as a consequence of bending. Since the gages lie at the same distance from the neutral axis and equal distances from the point of load application, they experience the same degree of bending stress. However, the fibers on the top of the beam are in tension while the fibers on the bottom are in compression.



**Figure 2.** Gages are mounted perpendicular to one another on opposite sides of the beam.

Ideally, the gages should have been mounted on the same side of the beam so that strain was measured at the same location. If they were both mounted on the top surface, for example, the longitudinal gage reading would be positive (tensile) while the lateral gage reading would be negative (compressive). However, when the lateral gage is mounted on the bottom surface, it is compressed in a direction parallel to the longitudinal axis of the beam. The Poisson's effect causes the gage to elongate along its axis resulting in a positive strain measurement. Thus, for Equation (7.3-2) to work, a negative sign must be associated with the lateral strain measurement. Then, by applying a known strain to the lateral gage and measuring the strain in the longitudinal gage, Poisson's ratio can be computed.

Strain gages are calibrated in uniaxial stress on a test specimen made of a given material. When the stress state or the material is different from that of the calibration specimen (which is nearly always the case), a correction should be made for transverse sensitivity (see Section 12.8). This can be done when the readings from two perpendicular gages are known. The approach taken here is to assume that the longitudinal gage is not adversely affected by transverse sensitivity. The reading in the lateral gage is adjusted by computing the ratio of the strains measured in the two perpendicular gages, and then using a correction factor gleaned from the chart included in Section 7.5.

A better alternative to correct for transverse sensitivity is to measure the apparent strains along the two perpendicular directions, say  $x$  and  $y$ , and compute the true strains in both gages using

$$\begin{aligned}\epsilon_{x_{true}} &= \frac{(1 - \nu_o) (\epsilon_{x_{measured}} - K_t \epsilon_{y_{measured}})}{1 - K_t^2} \\ \epsilon_{y_{true}} &= \frac{(1 - \nu_o) (\epsilon_{y_{measured}} - K_t \epsilon_{x_{measured}})}{1 - K_t^2}\end{aligned}\tag{7.3-7}$$

where  $K_t$  is the transverse sensitivity factor, and  $\nu_o$  is the Poisson's ratio of the beam used to calibrate  $K_t$ . Most strain gage manufacturers use a calibration beam having  $\nu_o = 0.285$ . Even though it is not required, the relations included in Equation (7.3-6) can be applied in this experiment. However, since the gages are positioned perpendicular to one another but subjected to equal and opposite stress conditions, the reading on the lateral gage must be considered negative.

## 7.4 Procedure

An aluminum beam, on which two strain gages are mounted, is used to determine Poisson's Ratio. The beam is loaded in bending to an arbitrary strain level. Since the stress state on the surface of the beam is uniaxial, the lateral and longitudinal strains can be measured to calculate Poisson's Ratio.

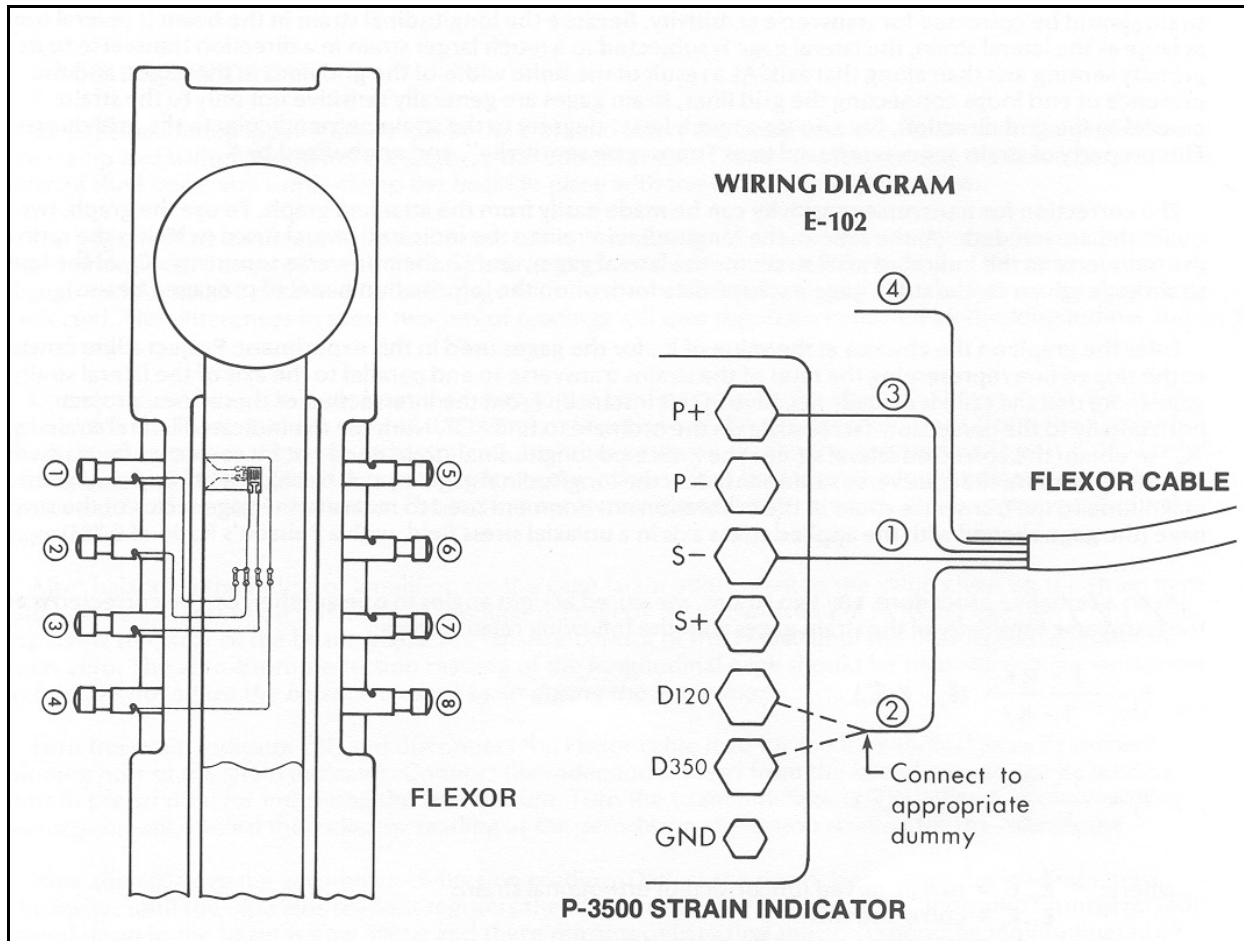
Information should be entered on the attached work sheet. The steps to be followed are:

1. From the beam, record the gage factor, ( $S_g$ ), and the transverse sensitivity, ( $K_t$ ).
2. Back the calibrated loading screw out of the way, and insert the beam into the Flexor with the gaged end in the clamp and with the longitudinal gage on the upper surface. Center the free end of the beam between the sides of the Flexor, and firmly clamp the beam in place with the knurled clamping screw.
3. The gages will be connected (via the Flexor cable) to the strain indicator one at a time - first, with the beam undeflected, and again with the beam deflected. An initial "reference" reading of the strain indicator digital readout will be obtained for each gage with the beam undeflected, and a final reading with the beam deflected. The differences in these two sets of readings will give the strain indicated by the longitudinal and lateral gages. **Although both strain readings are positive, the lateral strain is considered negative to take into account that the gages are subjected to equal but opposite stress conditions.**

Referring to Figure 3, connect the lead wires from the longitudinal strain gage to the posts on the sides of the "flexor" frame. As illustrated in the diagram, connect the appropriate gage leads from the Flexor cable to the S-, P+, and D-120 binding posts of the P-3500 strain indicator. Note: The strain gages employed in this experiment employ a "quarter-bridge" arrangement, since the lead labeled as 2 is connected to the D120 post on the P-3500.

4. Using Section 12.9 for guidance, depress the Amp Zero button and balance the amplifier. Then depress the Gage Factor button and set the gage factor (as displayed in the LCD readout) to the value provided by the manufacturer on the cantilever beam. Select the X1 MULT position and depress the RUN push button. With the Flexor loading screw still clear of the beam, adjust the balance control of the P-3500 until the LED digital readout indicates zero. The zero-beam-deflection reading of the longitudinal gage should be recorded on the worksheet as 0  $\mu\epsilon$ . Do not adjust the balance control again during the experiment.





**Figure 3.** Wiring diagrams for determination of Poisson's ratio.

5. Turn the strain indicator off, and disconnect the Flexor cable lead for the longitudinal gage from the P+ binding post of the strain indicator. Connect the independent lead from the lateral gage to the P+ binding post in preparation for indicating the lateral strain. Turn the strain indicator on (RUN) and, without making any adjustment, record the indicator reading as the zero-beam-deflection reading for the lateral gage.
6. Now add  $500 \mu\epsilon$  to the zero-beam-deflection reading. (This value is chosen to provide a load sufficient to obtain strain readings, but without exceeding the yield stress of the material.) Deflect the beam, by rotating the loading screw clockwise, until the indicator readout registers the number equal to this sum. The "indicated" (uncorrected) lateral strain in the beam is now  $500 \mu\epsilon$  and there remains only to find the corresponding longitudinal strain.
7. Turn off the strain indicator, disconnect the independent lateral-gage lead from the P+ binding post, and reconnect the longitudinal gage to the instrument. Turn the indicator on

again and record the number registered by the indicator readout. This number is equal to the longitudinal strain, since the balance was set to zero when the gage was first installed. If the circuit was not perfectly balanced, it would have been necessary to subtract the initial strain reading from this number to obtain the longitudinal strain. Poisson's Ratio is then  $500 C/X$ , where "C" is the correction factor for the transverse sensitivity of the lateral gage, and "X" is the second reading of the longitudinal gage (details follow).

8. As a check on the stability of the system, back the Flexor loading screw away until it clears the beam. The strain indicator readout should now read very close to  $0 \mu\epsilon$  if the system is operating normally. If the number is more than  $\pm 10 \mu\epsilon$  or so, the source of the error should be located, and the experiment performed again.

## 7.5 Laboratory Report

### FIGURES:

Include a sketch of the beam, indicate the support and load application point. Clearly indicate the location and orientation of the gages.

### CALCULATIONS:

1. Find the indicated strains for the lateral and longitudinal gages by subtracting the undeflected readings from the deflected values. For the lateral gage, this will be  $500 \mu\epsilon$ . However, this value will be considered negative to take into account that the gages are mounted on opposite sides of the specimen and subjected to equal and opposite stress conditions.
2. Correct the lateral strain for transverse sensitivity by using the graph included as Figure 4. This is accomplished by calculating the ratio of the apparent strains (*i.e.*, measured):

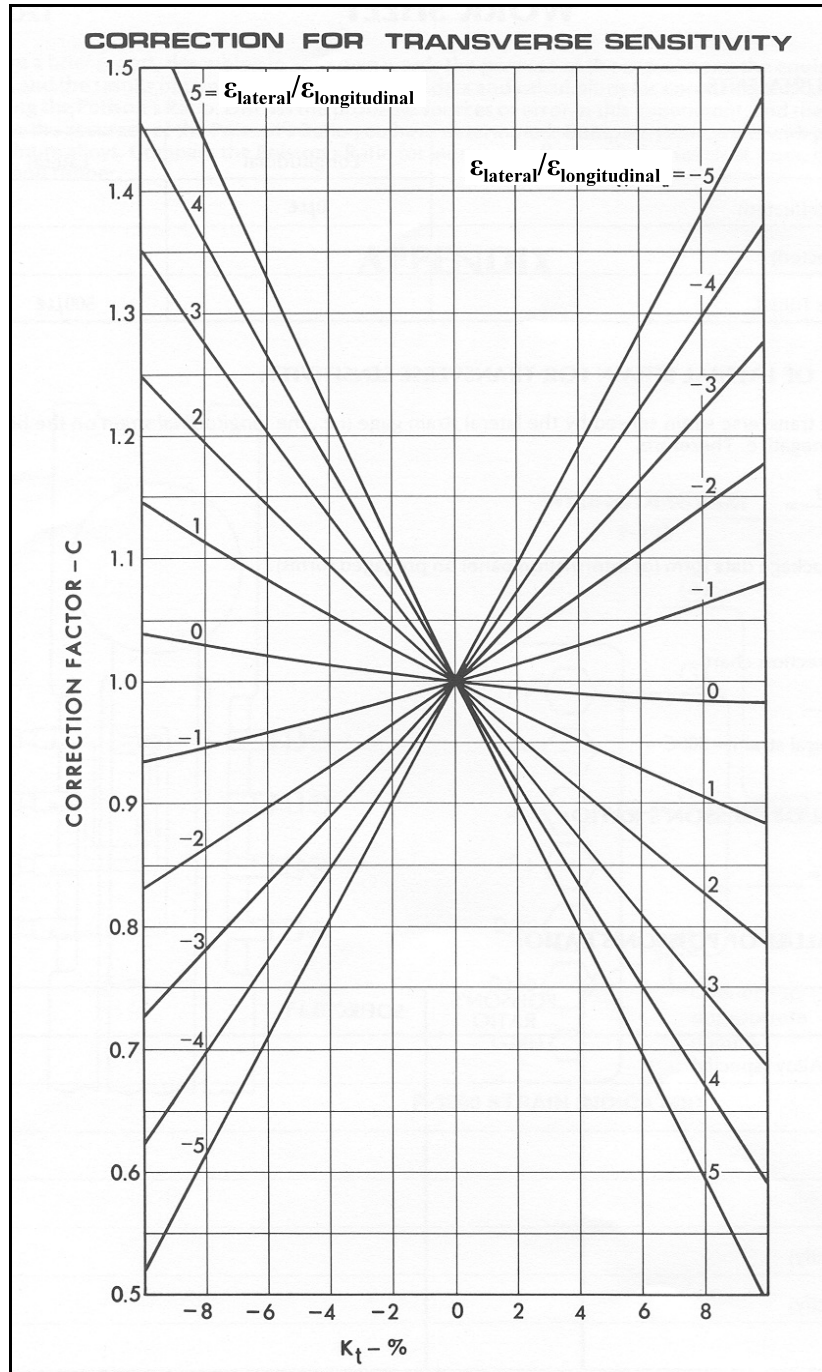
$$\frac{\hat{\epsilon}_{lateral}}{\hat{\epsilon}_{longitudinal}} = - \frac{lateral \ strain \ measurement}{longitudinal \ strain \ measurement} \quad (5.5-1)$$

Remember to include the negative sign in Equation (7.5-1) to reflect the fact that the signs of the strains would have been opposite if the gages were placed perpendicular to one another on the same side of the specimen. Now locate  $K_t$ , recorded in Step 1 of the procedure, on the abscissa of the graph. Project a line upward to the sloped line corresponding to the ratio determined in Equation (7.5-1). Project a horizontal line from the ordinate scale to find the correction factor, "C".

3. Calculate Poisson's Ratio using  $500 C/X$ , where "C" is the correction factor for the transverse sensitivity of the lateral gage, and "X" is the second reading taken from the longitudinal gage

(i.e., the numerator is the corrected lateral strain while the denominator is the net strain measured by the longitudinal strain gage).

4. Include a calculation of percentage error from the standard value for aluminum in your report. Discuss possible error sources.



**Figure 4.** Transverse sensitivity correction chart.

## WORK SHEET FOR POISSON'S RATIO

GAGE FACTOR ( $S_g$ ): \_\_\_\_\_

TRANSVERSE SENSITIVITY FACTOR ( $K_t$ ): \_\_\_\_\_

STRAIN MEASUREMENTS:

	LONGITUDINAL ( $\mu\epsilon$ )	LATERAL ( $\mu\epsilon$ )
Undeflected	0	
Deflected		
Net Strain		- 500 <sup>1</sup>

CALCULATIONS FOR POISSON'S RATIO:

$$\hat{\epsilon}_{longitudinal} = \hat{\epsilon}_{longitudinal (deflected)} - \hat{\epsilon}_{longitudinal (undeflected)} = \text{_____} \mu\epsilon$$

$$\hat{\epsilon}_{lateral} = - 500 \mu\epsilon \quad (\text{See footnote below.})$$

$$\frac{\hat{\epsilon}_{lateral}}{\hat{\epsilon}_{longitudinal}} = \frac{- 500}{(\quad)} = - \text{_____} \quad (\text{This value should be negative.})$$

$$C = \text{_____}$$

$$\nu = - \frac{(- 500) C}{\hat{\epsilon}_{longitudinal}} = \text{_____}$$

$$\nu_{standard} = 0.31$$

$$\% \text{ error} = \frac{\nu_{standard} - \nu_{calculated}}{\nu_{standard}} \times 100 = \text{_____}$$

---

<sup>1</sup>For the purposes of correcting for transverse sensitivity and computing Poisson's ratio, a negative sign has been assigned to the lateral strain. This was done to reflect the fact that the signs of the strains would have been opposite if the gages were placed perpendicular to one another on the same side of the specimen.

## CHAPTER 8 - CANTILEVER FLEXURE TEST

### 8.1 Objective

The purpose of this experiment is to perform a detailed analysis of a cantilever beam. During the experiment students will: (1) determine the applied load and shear force from strain measurements, (2) verify the linearity of strain along the longitudinal axis of the beam, and (3) confirm the shear force and moment relationships by comparing two different stress determinations. The experiment should take approximately 45 minutes to run; a maximum group size of 4 people is recommended.

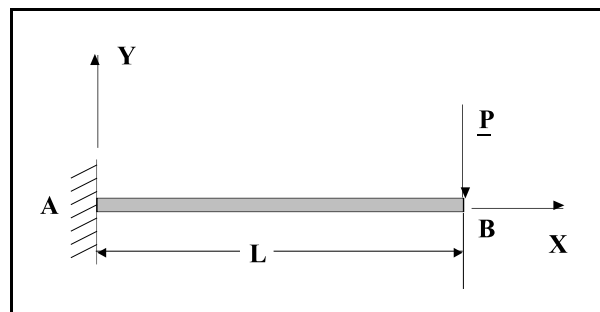


### 8.2 Materials and Equipment

- Cantilever flexure frame (see Section 12.6 for details)
- No. B105 (2024-T6 high-strength aluminum alloy beam); 1/4 x 1 x 12.5 in. (6x25x320 mm)
- P-3500 strain indicator (see Section 12.9 for details)
- Micrometer (see Section 12.1 for details)

### 8.3 Background

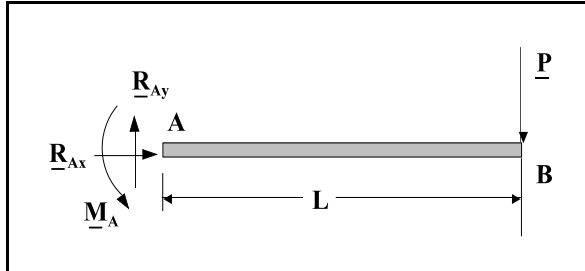
The designation "cantilever" is commonly applied to any beam which is built-in and supported at only one point; the beam is loaded by one or more point loads or distributed loads acting perpendicular to the beam axis. Figure 1, for example, shows a cantilever beam of length,  $L$ , fixed into a wall at the left end and subjected to a concentrated load,  $P$ , applied at the right end.



**Figure 1.** An end-loaded cantilever beam.

Even though the beam in Figure 1 is a very simple example of a cantilever, understanding this case is important because the basic configuration is widely employed during construction of relatively complex structural elements. In practice, a cantilever beam may vary in section along its length, the mounting details at the fixed end may differ greatly, and the system of applied loads can take a variety of forms. Examples include airplane wings, supports for overhanging roofs, and gear teeth.

Figure 2 shows the free body diagram corresponding to the beam illustrated in Figure 1. Since all of the forces are contained in a single plane, the reactions at the fixed support consist of two force reactions and one moment reaction. Referring to Figure 2, and taking into account overall equilibrium,

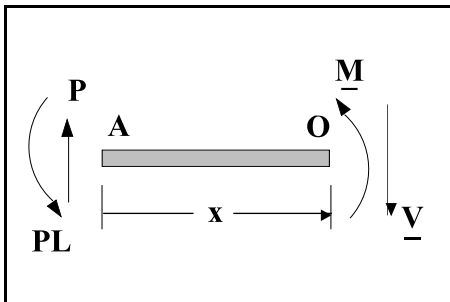


**Figure 2.** FBD of the cantilever beam.

$$\sum F_x = 0 = R_{Ax} \quad R_{Ax} = 0 \quad (8.3-1)$$

$$\sum F_y = 0 = R_{Ay} - P \quad R_{Ay} = P \quad (8.3-2)$$

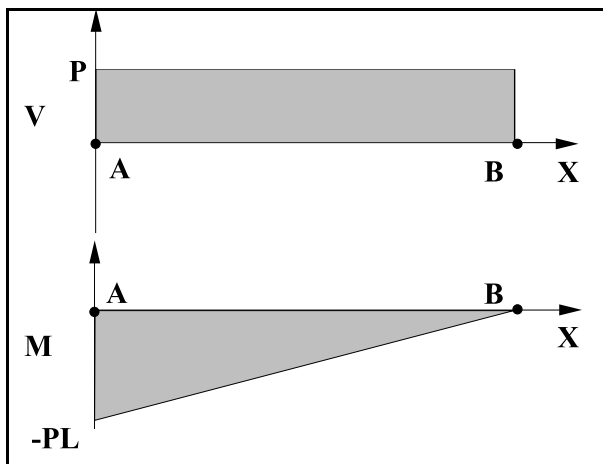
$$\sum M_A = 0 = M_A - P L \quad M_A = P L \quad (8.3-3)$$



**Figure 3.** A cut at position x.

Expressions for the shear force, V, and bending moment, M, can be obtained for all points along the span by passing a section through a point, O, located at an arbitrary distance, x, from the fixed support. Figure 3 shows this procedure and illustrates the standard convention for assigning the shear and moment distribution on the cut section. Applying the equilibrium equations to the section:

$$\sum F_y = 0 = -V + P \quad V = P \quad (8.3-4)$$



**Figure 4.** Shear and moment diagrams.

$$\sum M_O = 0 = M + P L - P x \quad (8.3-5)$$

$$M = -P(L - x)$$

The distance between the point O and the load is often referred to as effective length,  $L_e$ , of the beam.

Equations (8.3-4) and (8.3-5) can be used to plot the shear and moment diagrams shown in Figure 4. The shear force is constant over the span and equals the applied load. The bending moment is maximum at the fixed support and progressively

decreases as the effective length becomes smaller.

The diagrams are consistent with the fact that the shear force at each section is equal to the derivative of the bending moment (the slope of the moment diagram). Conversely, the difference in the bending moment between two sections is equal to the area under the shear curve. That is, for small deflections,

$$V = \frac{dM}{dx} . \quad (8.3-6)$$

The shear force creates a shear stress at every section that can be approximated by dividing the force by the cross-sectional area. However, this approximation only gives an average value for the stress and does not adequately define the actual distribution through the thickness. A better description is given by,

$$\tau = \frac{V Q}{I t} \quad (8.3-7)$$

where  $V$  is the shear force,  $Q$  is the first moment of the area measured about the neutral axis of the portion of the cross section located either above or below the point under consideration,  $I$  is the centroidal moment of inertia of the entire cross section, and  $t$  is the width of a cut made through the point in question perpendicular to the applied load.

When Equation (8.3-7) is applied to a beam of rectangular cross section, it is found that the shear stress distribution is parabolic through the thickness. The shear stress is zero at the upper and lower surfaces of the beam and maximizes at the neutral axis.

Shear stresses always come in pairs, and it should be noted that the vertical shear force creates a horizontal shear flow. That is, horizontal shear stresses develop between the longitudinal fibers in the beam. This effect is zero on the free surface and maximum at the neutral axis.

The bending moment creates normal stresses parallel to the longitudinal axis of the beam. The magnitude of these stresses is given by the elastic flexure formula,

$$\sigma = - \frac{M y}{I} \quad (8.3-8)$$

where  $M$  is the moment,  $y$  is the distance measured from the neutral axis to the point under consideration, and  $I$  is the centroidal moment of inertia corresponding to the axis about which the moment is applied.

Equation (8.3-8) shows that the normal stress is linearly distributed through the thickness. It

becomes a maximum at the free surfaces and reduces to zero at the neutral axis.

Equation (8.3-8) can be applied to determine the stress on the upper surface of the cantilever beam shown in Figure 1. Let us assume that the beam has width,  $b$ , and thickness,  $t$ . Since the neutral axis is located at the geometric center of the beam,  $y = b/2$ . The centroidal moment of inertia measured around the bending axis ( $z$ ) is  $I = 1/12 bt^3$ ; the moment is given as a function of  $x$  by Equation (8.3-5). Substituting these values into Equation (8.3-8),

$$\sigma_x = - \frac{- P (L - x) \frac{b}{2}}{\frac{1}{12} b t^3} = \frac{6 P (L - x)}{b t^2} . \quad (8.3-9)$$

Equation (8.3-9) shows that the normal stress at the top of the section is tensile. Since the fibers on this surface do not experience any shear stress, the points located there are in a state of uniaxial stress. In this simple case, the constitutive relations (Hooke's Laws) reduce to,

$$\sigma_x = E \epsilon_x \quad (8.3-10)$$

where  $E$  is the elastic modulus and  $\epsilon_x$  is the normal strain measured along the longitudinal axis of the beam.

Substituting Equation (8.3-9) into (8.3-10) and solving for the strain,

$$\epsilon_x = \frac{6 P (L - x)}{E b t^2} \quad (8.3-11)$$

Equation (8.3-11) implies that the strain along the beam varies linearly with  $x$ . Consequently, a plot of strain versus distance should result in a straight line.

Equation (8.3-11) can also be differentiated with respect to  $x$  in order to establish a relation between the applied load and the difference in strain between points located at different  $x$  values as follows:

$$\frac{\Delta \epsilon_x}{\Delta x} = - \frac{6 P}{E b t^2} . \quad (8.3-12)$$

Solving Equation (8.3-12) for  $P$ ,

$$P = - \frac{E b t^2}{6} \frac{\Delta \epsilon_x}{\Delta x} . \quad (8.3-13)$$



Equation (8.3-4) reveals that the load equals the shear force at every location. Thus, Equation (8.3-13) shows that when two strain gages are positioned at difference locations along the span, the shear force is directly proportional to the difference in their strain readings.

Equation (8.3-13) can be applied to the case in which three strain gages are positioned along the span. In this case,

$$P_{1,2} = - \frac{E b t^2}{6} \frac{(\epsilon_1 - \epsilon_2)}{(x_1 - x_2)} \quad \text{and/or} \quad P_{2,3} = - \frac{E b t^2}{6} \frac{(\epsilon_2 - \epsilon_3)}{(x_2 - x_3)}. \quad (8.3-14)$$

The answers extracted from these expressions will most likely differ slightly because of experimental error so their average can be used as the best estimate of the load. According to Equation (8.3-4), the average should also be equal to the shear force.

Another method of calculating the load is to plot the individual strain readings ( $\epsilon_x$ ) versus the distance from the fixed support ( $x$ ). After drawing the best straight line through the data, the slope of the line,  $\Delta\epsilon_x/\Delta x$ , can be used in Equation (8.3-13) to confirm the load calculated from the average of the expressions included as Equation (8.3-14). These values should be very close.

The load can also be obtained by knowing the deflection under the load. The governing expression can be found by applying the equation,

$$\frac{d^2y}{dx^2} = \frac{M(x)}{E I} \quad (8.3-15)$$

where  $M(x)$  is the expression for the applied moment given by Equation (8.3-5). Thus,

$$E I \frac{d^2y}{dx^2} = - P L + P x . \quad (8.3-16)$$

Integrating once,

$$E I \frac{dy}{dx} = - P L x + \frac{P x^2}{2} + C_1 . \quad (8.3-17)$$

Integrating again,

$$E I y = - \frac{P L x^2}{2} + \frac{P x^3}{6} + C_1 x + C_2 . \quad (8.3-18)$$

Applying the boundary conditions [ $@ x = 0, dy/dx = 0$ ] and [ $@ x = 0, y = 0$ ] to the expressions in

Equations (8.3-17) and (8.3-18), respectively, reveals that  $C_1 = C_2 = 0$ . Therefore,

$$y = \frac{P}{6 E I} ( x^3 - 3 L x^2 ) . \quad (8.3-19)$$

Under the load at  $x = L$ ,

$$y = - \frac{P L^3}{3 E I} . \quad (8.3-20)$$

Denoting the absolute value of this deflection by  $\delta_{end}$ , and solving for P,

$$P = \frac{3 \delta_{end} E I}{L^3} . \quad (8.3-21)$$

In summary:

The applied load, P, which is equal to the shear force along the beam, can be calculated by (1) making differential strain measurements and averaging the values obtained from the included in Equation (8.3-14); (2) plotting the individual strain measurements, finding the slope,  $\Delta \epsilon_x / \Delta x$ , or the best straight line through the data and applying Equation (8.3-13); or, measuring the deflection under the load and applying Equation (8.3-21).

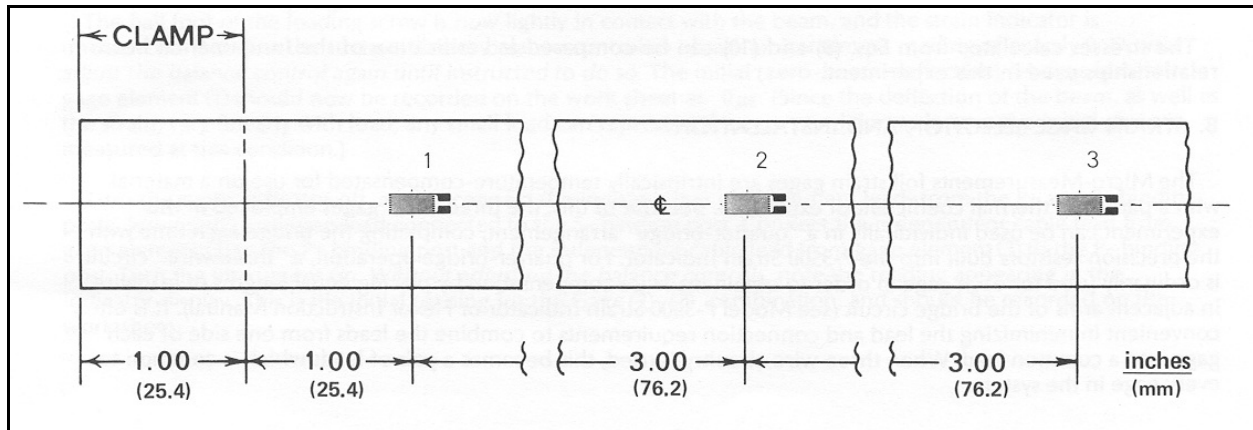
The stress at any point on the top surface of the beam can be found by: (1) using the flexure formula in Equation (8.3-9) with the value for P obtained above; or, measuring strain at that point and using Equation (8.3-10).

## 8.4 Procedure

The experiment is performed by using a beam with three strain gages mounted at uniform intervals along the axis of the beam. This configuration is illustrated in Figure 5.

Information should be entered on the attached work sheet. The steps to be followed are:

1. Back the calibrated loading screw out of the way, and insert the beam into the Flexor with the gages on the upper surface. Center the free end of the beam between the sides of the Flexor, and firmly clamp the beam in place with the knurled clamping screw.

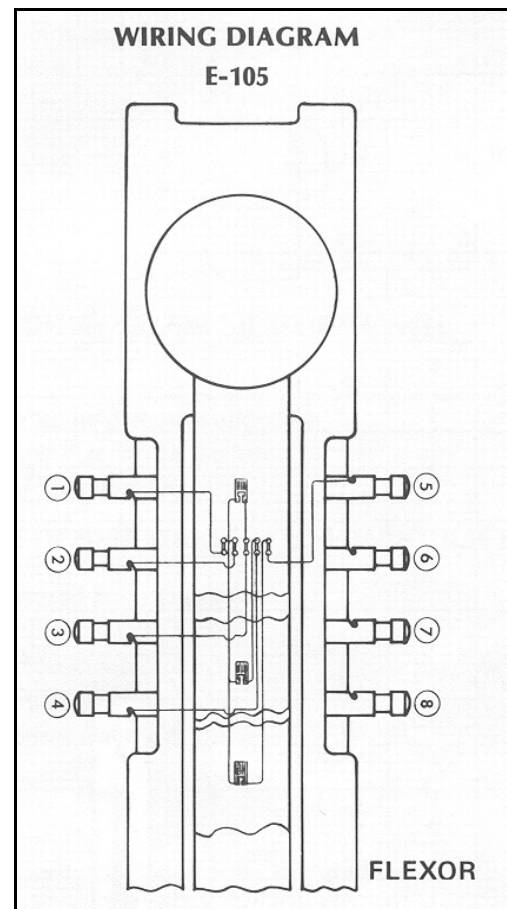


**Figure 5.** Three strain gages are mounted at uniform intervals along the span.

2. Measure and record the beam width ( $b$ ), beam thickness ( $t$ ), and the distances from the centerline of each gage to the fixed support.
3. Connect the lead wires from the strain gages to the binding posts of the flexure frame according to the wiring diagram shown in Figure 6.

The experiment is divided into two sections. The first entails taking differential strain readings, *i.e.*, the difference in strain between two gages. This utilizes the "half-bridge" arrangement of the Wheatstone bridge shown in Figure 7. With the differential strains known, the expressions in Equation (8.3-14) can be used directly to obtain the applied load.

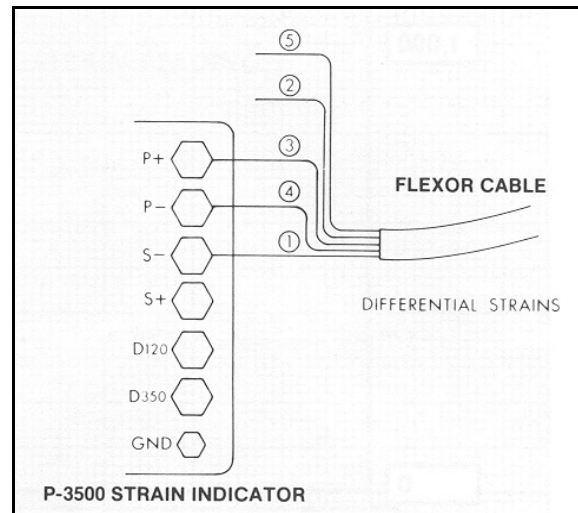
The second part of the experiment relies on taking individual strain readings for each gage using the quarter-bridge arrangement shown in Figure 8. From these measurements a plot of strain vs. distance from load can be used in Equation (8.3-13) to obtain the applied load.



**Figure 6.** Flexor wiring diagram.

## Part I: Differential Strain Readings

1. Follow the wiring diagram for differential strain measurement shown in Figure 7. Connect one of the common cable leads to the S- binding post of the strain indicator, and the independent lead from Gage(1) to the P+ binding post. Connect the independent lead from Gage (2) to the P- binding post. The two gages are now connected in adjacent arms of the bridge circuit to form a half-bridge configuration. By installing the gages in this manner, the strain indicated by the instrument is equal to the algebraic difference between the strains of the two gages. With this arrangement, the bridge-completion resistor (DI20) is not needed.



**Figure 7.** Half-bridge circuit for making differential strain measurements.

2. Using Section 12.9 for guidance, depress the Amp Zero button and balance the amplifier. Then set the gage factor adjustment to the value given on the beam. Select the X1 MULT position and depress the RUN push button. With the Flexor loading screw still clear of the beam, adjust the balance control of the P-3500 until the LCD digital display indicates 0.
3. Turn the Flexor loading screw clockwise until the ball foot comes into contact with the beam and the instrument readout indicates a small value of strain, say, 100  $\mu\epsilon$ . Note that the vertical displacement of the loading screw is 0.025 in (0.64 mm) per revolution. Record the initial micrometer reading on the data sheet as  $\delta_{\text{initial}}$ .

The ball foot of the loading screw is now lightly in contact with the beam, and the strain indicator is unbalanced. Readjust the strain indicator balance control to obtain an instrument reading of exactly 0. Do not adjust the balance control again until instructed to do so.

The initial (zero-beam-deflection) reading for  $(\epsilon_1 - \epsilon_2)$  should now be recorded on the worksheet as 0  $\mu\epsilon$ . (Since the deflection of the beam, as well as the strain, vary linearly with load, any small load can represent the zero condition as long as the initial strain is measured at this condition.)

4. Turn the strain indicator off, and disconnect the independent gage element leads from the P+ and P- binding posts, leaving the common lead connected to the S- binding post. Connect the independent cable lead from gage element (2) to the P+ binding post and the independent cable lead from gage element (3) to the P- binding post. Turn the instrument on. Without adjusting the balance controls, note the reading appearing in the indicator display. This is

the initial reading for  $(\epsilon_2 - \epsilon_3)$ , and should be recorded on the work sheet.

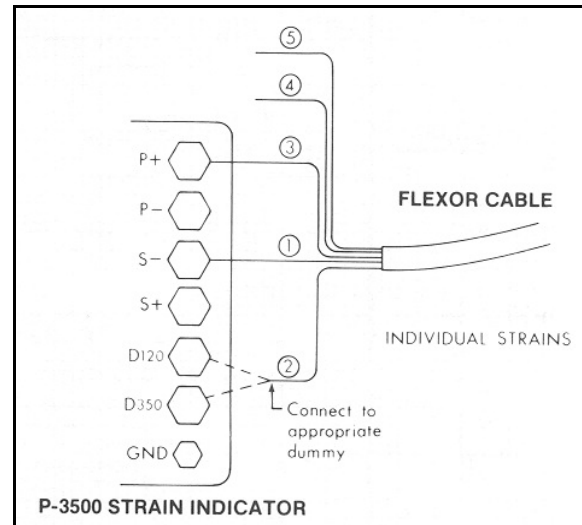
- After obtaining and recording the initial reading for  $(\epsilon_2 - \epsilon_3)$ , add  $600 \mu\epsilon$  to the value of this initial reading. (The strain increment is chosen to insure that enough load is applied to the beam so that strains can be measured using the P-3500, but without exceeding the yield stress of the material.) With Gages (2) and (3) connected, deflect the beam, by rotating the loading screw clockwise, until the indicator readout registers the number equal to this sum. Record this number in the appropriate place on the work sheet as the final reading for  $(\epsilon_2 - \epsilon_3)$ . Record the final micrometer reading on the data sheet as  $\delta_{\text{final}}$ .
- With the loading screw unchanged, turn off the strain indicator and disconnect the independent leads for Gages (2) and (3) from the instrument. Reconnect Gages (1) and (2) in the same manner as before. Turn the instrument on again and note the number registered by the indicator readout. This number is the final reading for  $(\epsilon_1 - \epsilon_2)$ , and should be recorded in the appropriate location on the work sheet. Note that except for experimental inaccuracies, the difference between the final and initial readings for Gages (1) and (2) should be  $600 \mu\epsilon$  as it was for Gages (2) and (3).

**Leave the beam deflected for Part II.** This insures the same beam deflection for both the differential strain measurements and the individual strain measurements.

## Part II. Individual Strain Readings

The gages will be connected (via the Flexor cable) to the strain indicator one at a time - first, with the beam deflected, and again with the beam undeflected. The strain indicator will be used to obtain readings from each gage for these two loading conditions; the difference between the final and initial strain readings provide a measure of the longitudinal strain in each gage.

- With the beam still deflected and the meter off, disconnect the Gage (2) independent lead from the P- binding post, leaving Gage (1) connected to the instrument. Referring to Figure 8, connect the second common lead from the Flexor to the DI20 post. Turn the instrument on and adjust the balance control until the LCD readout indicates precisely 0. Record the initial strain indication for Gage (1) as  $0 \mu\epsilon$  on the work sheet.



**Figure 8.** Quarter-bridge circuit for making individual strain measurements.

This is the new position of the balance control, and it should not be changed during the

remainder of the experiment. The P-3500 is now in a quarter-bridge configuration, designed to measure the strain in a single gage.

2. Turn the indicator off, disconnect the independent lead from Gage (1) from the P+ binding post, and connect the independent lead from Gage (2) to the P+ binding post. Turn the instrument on and without adjusting the balance control note the initial reading for Gage (2). Record this number in the appropriate location on the work sheet.
3. Repeat Step 2 for Gage (3), and record the initial reading on the work sheet for Gage (3).
4. With Gage (3) connected to the instrument, unload the beam until the micrometer reads the value originally recorded as the initial deflection ( $\delta_{\text{initial}}$ ). Note the new reading on the strain indicator display. This is the final reading for Gage (3) and should be recorded on the work sheet.
5. Turn the meter off. Disconnect Gage (3), reconnect Gage (2). Find and record the final (undeflected) strain reading for Gage (2).
6. Repeat Step 5, using Gage (1). Turn the meter off.

The strain readings for all three gages should decrease from the initial readings, since the load has been decreased.

## 8.5 Laboratory Report

### FIGURES:

Include a sketch of the beam noting dimensions, location and orientation of gages, and the location of the load.

### CALCULATIONS:

1. Using the differential strain measurements from Part I, apply the expressions in Equation (8.3-14) obtain the load, P. The average value obtained from the two expressions should be used, since this will give the best estimate.
2. Find the individual strains for the three gages by subtracting the final readings from the initial readings of the data obtained in Part II.
3. Plot these strains ( $\epsilon_x$ ) versus the distance measured from the fixed support (x). Draw the "best" straight line through the points. Measure the slope,  $\Delta\epsilon_x/\Delta x$ . (The slope should be negative since the strain decreases with x.)

4. Calculate  $P$  using the slope determined in Step 3 and Equation (8.3-13).
5. Calculate  $P$  from the measured deflection using Equation (8.3-21). First determine  $\delta_{\text{end}}$  by subtracting the initial deflection reading from the final. Calculate the moment of inertia, then solve for  $P$ .
6. Compare  $P$  obtained in Steps 1, 4, and 5.
7. Using  $P$  obtained in Step 1 and Step 5 calculate the stress at each gage using Equation (8.3-9).
8. Calculate the stress at each gage using the strains obtained in Part II and Hooke's Law [Equation (8.3-10)].
9. Compare the stresses obtained in Steps 7 and 8.
10. Using the load obtained from the deflection (Step 5), calculate the strain at each gage. Compare these values to the measured values from Part II. Discuss possible error sources.

## WORK SHEET FOR CANTILEVER FIXTURE

### BEAM DIMENSIONS:

b = \_\_\_\_\_ inches (width)  
t = \_\_\_\_\_ inches (thickness)  
L = \_\_\_\_\_ inches (from clamp to applied load)

### GAGE LOCATIONS:

$x_1$  = \_\_\_\_\_ inches (distance from clamp to centerline of Gage 1)  
 $x_2$  = \_\_\_\_\_ inches (distance from clamp to centerline of Gage 2)  
 $x_3$  = \_\_\_\_\_ inches (distance from clamp to centerline of Gage 3)

### PART I - DIFFERENTIAL STRAIN MEASUREMENTS:

	$\epsilon_1 - \epsilon_2$ ( $\mu\epsilon$ )	$\epsilon_2 - \epsilon_3$ ( $\mu\epsilon$ )
Initial ("zero" deflection)	0	
Final (maximum deflection)		
Final minus initial		600

$\delta_{\text{initial}}$  = \_\_\_\_\_ inches (initial deflection)       $\delta_{\text{final}}$  = \_\_\_\_\_ inches (final deflection)

### PART II - INDIVIDUAL STRAIN MEASUREMENTS:

	$\epsilon_1$ ( $\mu\epsilon$ )	$\epsilon_2$ ( $\mu\epsilon$ )	$\epsilon_3$ ( $\mu\epsilon$ )
Initial (maximum deflection)	0		
Final ("zero deflection")			
Initial minus final (net $\epsilon$ ) <sup>1</sup>			

---

<sup>1</sup> The final reading is subtracted from the initial reading to obtain the net strain because the beam is deflected when the initial reading is recorded.



CALCULATIONS FOR CANTILEVER FIXTURE:

Use  $E = 10.4 \times 10^6$  psi; remember  $\mu\epsilon = \epsilon \times 10^{-6}$ .

1. LOAD ESTIMATE 1:

$$P_{1,2} = - \frac{E b t^2}{6} \frac{(\epsilon_1 - \epsilon_2)}{(x_1 - x_2)}$$

$$P_{2,3} = - \frac{E b t^2}{6} \frac{(\epsilon_2 - \epsilon_3)}{(x_2 - x_3)}$$

$$P = \frac{P_{1,2} + P_{2,3}}{2} = \text{_____ } lb$$

2. INDIVIDUAL STRAINS:

$$\epsilon = \epsilon_{initial} - \epsilon_{final}$$

$$\epsilon_1 = \text{_____} - \text{_____} = \text{_____ } \mu\epsilon$$

$$\epsilon_2 = \text{_____} - \text{_____} = \text{_____ } \mu\epsilon$$

$$\epsilon_3 = \text{_____} - \text{_____} = \text{_____ } \mu\epsilon$$

3. STRAIN GRADIENT:

(include graph) slope,  $\frac{\Delta\epsilon_x}{\Delta x} = \text{_____} \frac{\mu\epsilon}{in.}$

4. LOAD ESTIMATE 2:

$$P = - \frac{E b t^2}{6} \frac{\Delta\epsilon_x}{\Delta x} .$$

5. LOAD ESTIMATE 3:

$$\delta_{end} = \delta_{final} - \delta_{initial}$$

$$I = \frac{b t^3}{12} = \text{_____ } in.^4$$

$$P = \frac{3 \delta_{end} E I}{L^3} = \text{_____ } lb$$

6. LOAD COMPARISONS FOR METHODS 1-3:

Differential strain measurement (lb)	Slope through individual strain readings (lb)	End deflection (lb)

7. STRESS ESTIMATE 1:

$$\sigma_x = \frac{6 P (L - x)}{b t^2}$$

Station	(L - x); (in inches); from fixed support	$\sigma_x$ (psi) using P from Step 1	$\sigma_x$ (psi) using P from Step 5
1			
2			
3			

8. STRESS ESTIMATE 2:

$$\sigma_x = E \epsilon_x$$

Station	x (in.)	$\sigma$ (psi)
1		
2		
3		

9. STRESS COMPARISON FOR METHODS 1 AND 2:

	$\sigma_1$ (psi)	$\sigma_2$ (psi)	$\sigma_3$ (psi)
Using P from Step 1			
Using P from Step 5			
From Hooke's Law			

10. STRAIN CALCULATIONS:

$$\text{Using } P \text{ from Step 5} \quad \text{and} \quad \epsilon_x = \frac{6 P (L - x)}{b t^2 E}$$

	$\epsilon_1$ ( $\mu\epsilon$ )	$\epsilon_2$ ( $\mu\epsilon$ )	$\epsilon_3$ ( $\mu\epsilon$ )
Calculated			
Measured			
Errors			

## CHAPTER 9 - STRESS CONCENTRATION

### 9.1 Objective

The purpose of this experiment is to demonstrate the existence of stress and strain concentration in the vicinity of a geometric discontinuity in a cantilever beam, and to obtain an approximate measure of the elastic (theoretical or geometric) stress concentration factor,  $K_t$ . In this instance, the discontinuity is simply a circular hole, drilled through the depth of the beam on its centerline. The experiment should take approximately 25 minutes to run; a maximum group size of 4 people is recommended.



### 9.2 Materials and Equipment

- Cantilever flexure frame (see Section 12.6 for details)
- No. B104 (2024-T6 high-strength aluminum alloy beam); 1/4 x 1 x 12.5 in. (6x25 x320 mm)
- P-3500 strain indicator (see Section 12.9 for details)
- Micrometer (see Section 12.1 for details)

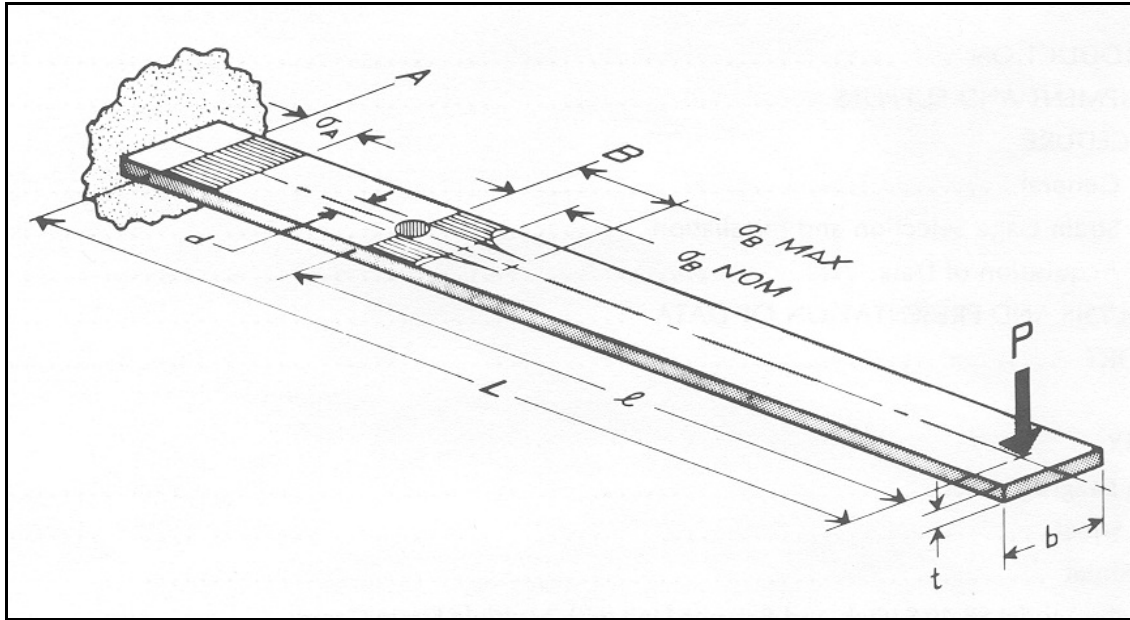
### 9.3 Background

The presence of any geometric irregularity in the shape of a loaded mechanical part or structural member impedes the orderly flow of stress trajectories, causing them to crowd together, and locally increasing the stress above the nominal level as calculated by conventional mechanics of materials formulas. Such an irregularity or discontinuity is referred to as a "stress-raiser". Figure 1, for example, shows the stress distribution at two sections of a cantilever beam, and illustrates the presence of stress concentration.

A theoretical value of the raised stress is found from the stress concentration factor,  $K_t$ . This factor is the ratio of the maximum stress at the discontinuity to the nominal stress at that point. Nominal stress is the stress based on the net area of the section. From the nominal stress and the stress concentration factor, the actual stress at the discontinuity can be estimated.

In the experiment at hand, it is convenient to measure the nominal stress at a section of the beam that is removed from the discontinuity but this location must be carefully selected so that the magnitude of this stress is the same as that computed based on the area of the discontinuity. The analysis

follows.



**Figure 1.** A geometrical discontinuity in a cantilever beam.

The stress distribution in the cantilever beam was studied in detail in Chapter 8. When the flexure formula was applied to determine the stress on the top surface of a beam having no discontinuity, it was found that

$$\sigma_x = - \frac{M y}{I} = - \frac{- P (L - x) \frac{b}{2}}{\frac{1}{12} b t^3} = \frac{6 P (L - x)}{b t^2} \quad (9.3-1)$$

where P is the applied load, L is the distance between the fixed support and the load, x is the coordinate measured to the point from the fixed support, b is the width, and t is the thickness of the beam. Assuming that the nominal stress is to be measured at Section A,

$$\sigma_x = \frac{6 P (L - x_A)}{b t^2} \quad (9.3-2)$$

At Section B, the cross sectional area must be reduced by taking into account the diameter of the hole, d. By applying the flexure formula to a point on the top surface of the beam,

$$\sigma_x = - \frac{M y}{I} = - \frac{- P (L - x_B) \frac{b}{2}}{\frac{1}{12} (b - d) t^3} = \frac{6 P (L - x_B)}{(b - d) t^2} \equiv \sigma_{nominal} \quad (9.3-3)$$

The condition under which the stresses in Equations (9.3-2) and (9.3-3) are equal is determined by equating the right hand sides of the equations, requiring that

$$\frac{L - x_B}{L - x_A} = \frac{b - d}{b} . \quad (9.3-4)$$

Since the condition presented in Equation (9.3-4) was satisfied when fabricating the cantilever beam to be tested, the stress measured at Section A, and the expression given in Equation (9.3-2), is the same as the nominal stress at the discontinuity located at Section B.

The stress at Section B, however, varies over the width due to the stress concentration effect. The maximum stress exists at the edge of the hole, on the transverse diameter, and the stress decreases rapidly with distance from the hole.

By definition, the stress concentration factor,  $K_t$ , is the ratio of the maximum stress at the stress-raiser to the nominal stress at the same point. That is,

$$K_t = \frac{\sigma_{B_{maximum}}}{\sigma_{B_{nominal}}} = \frac{\sigma_{B_{maximum}}}{\left[ \frac{6 P (L - x_B)}{(b - d) t^2} \right]} . \quad (9.3-5)$$

where  $\sigma_{nominal}$  is given by Equation (9.3-3).

Since the nominal stress at both sections of the beam and the peak stress at the edge of the hole are all uniaxial, the strain and stress are proportional if the proportional limit of the beam material is not exceeded in the experiment. Thus, Equation (9.3-5) may also be written in terms of strain as

$$K_t = \frac{\epsilon_{B_{maximum}}}{\epsilon_{B_{nominal}}} . \quad (9.3-6)$$

Since the beam was configured so that the nominal strain at B was equal to the strain at A,  $\epsilon_A$ ,

$$K_t = \frac{\epsilon_{B_{maximum}}}{\epsilon_A} . \quad (9.3-7)$$

The strain distribution adjacent to the hole can be approximated by an expression of the form,

$$\epsilon = A + B \left[ \frac{R}{Z} \right]^2 + C \left[ \frac{R}{Z} \right]^4 \quad (9.3-8)$$

where R is the radius of the hole, Z is the distance from the center of the hole to any point on the transverse centerline; and A, B, and C, are constant coefficients to be determined from the measured strains at three different points along the transverse centerline.

The beam is equipped with three gages positioned at  $Z_1$ ,  $Z_2$ , and  $Z_3$ , respectively. Thus,

$$\begin{aligned}\epsilon_1 &= A + B \left[ \frac{R}{Z_1} \right]^2 + C \left[ \frac{R}{Z_1} \right]^4 \\ \epsilon_2 &= A + B \left[ \frac{R}{Z_2} \right]^2 + C \left[ \frac{R}{Z_2} \right]^4 \\ \epsilon_3 &= A + B \left[ \frac{R}{Z_3} \right]^2 + C \left[ \frac{R}{Z_3} \right]^4\end{aligned}\tag{9.3-9}$$

where  $R = 0.125$  in (3.18 mm);  $Z_1 = 0.145$  in (3.68 mm) ;  $Z_2 = 0.185$  in (4.70 mm), and  $Z_3 = 0.325$  in (8.26 mm). Solving the expressions in Equation (9.3-9) simultaneously for the constants,

$$\begin{aligned}C &= 5.86 (\epsilon_1 - \epsilon_2) - 5.44 (\epsilon_2 - \epsilon_3) \\ B &= 3.49 (\epsilon_1 - \epsilon_2) - 1.20 C \\ A &= \epsilon_1 - 0.743 B - 0.552 C\end{aligned}\tag{9.3-10}$$

Since  $R/Z = 1$  at the edge of the hole,

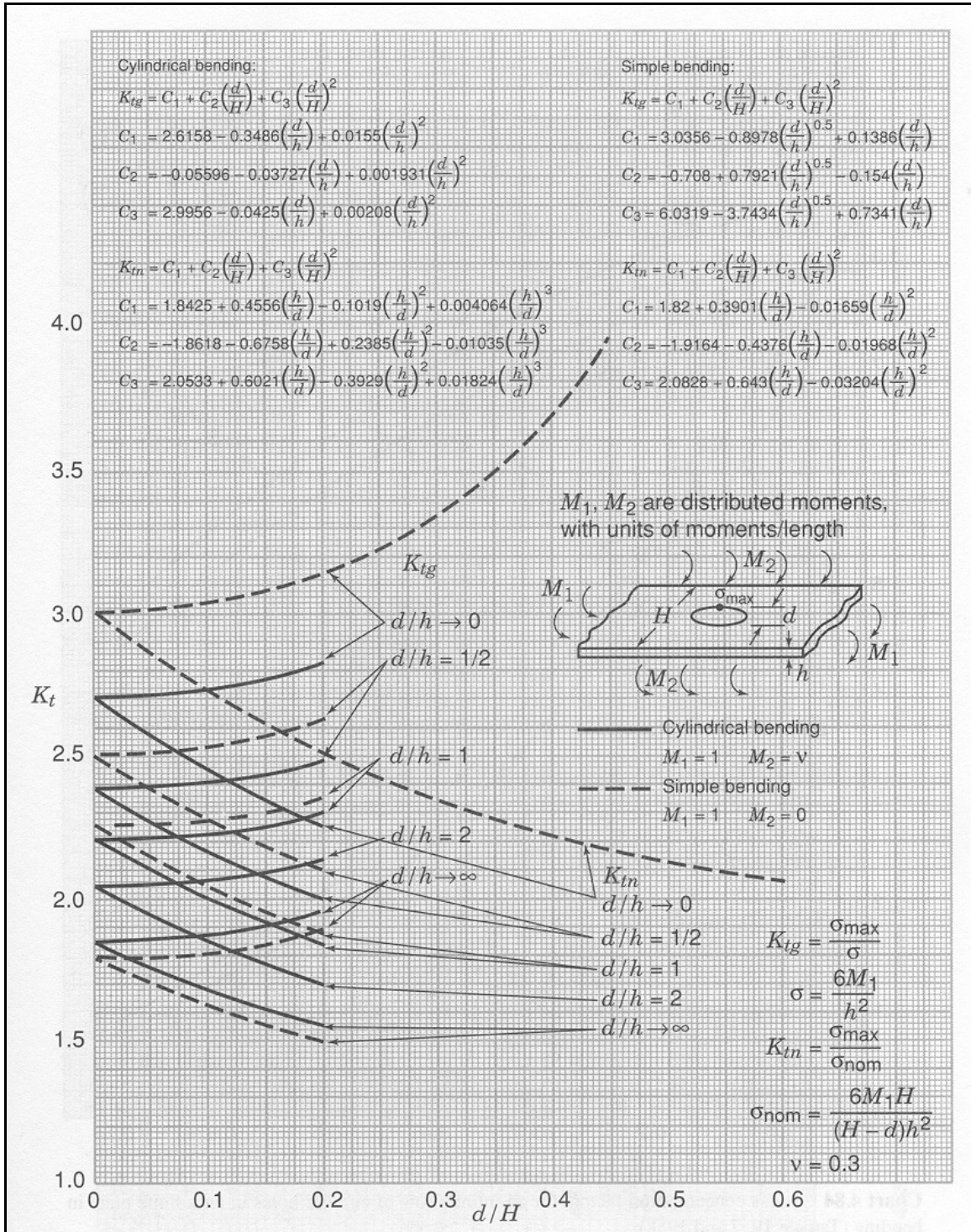
$$\epsilon_{B_{maximum}} = A + B + C\tag{9.3-11}$$

and

$$K_t = \frac{\epsilon_{B_{maximum}}}{\epsilon_{4_{corrected}}}\tag{9.3-12}$$

where  $\epsilon_4$  is the strain measured at Section A by using the correct gage factor.

A comparison of this experimental value can be made to the standard value determined from the chart included as Figure 2. Note that the ratio of the hole diameter to beam width and the ratio of hole diameter to the beam thickness is required to find  $K_t$  on the chart.



**Figure 2.** Stress concentration factors for bending of a finite-width plate with a circular hole. Chart taken from: Pilkey, W.D., *Peterson's Stress Concentration Factors*, Wiley and Sons, New York, 1997, p. 359.



## 9.4 Procedure

1. Back the calibrated loading screw out of the way, and insert the beam into the Flexor with the gaged end in the clamp, and with the gages on the top surface. Center the free end of the beam between the sides of the Flexor, making certain that the end of the beam is inserted into the clamp as far as it will go, and firmly clamp the beam in place with the knurled clamping screw.
2. Measure and record the beam width (b), beam thickness (t), diameter of the hole (d), distance from the centerline of the gages to the fixed support (x), distance between the fixed support and the load (L), and the gage factors for the gages as listed on the beam ( $S_{g1}$ ,  $S_{g2}$ ,  $S_{g3}$ ,  $S_{g4}$ ).

The gages will be connected (via the Flexor cable) to the strain indicator one at a time, first with the beam undeflected, and again with the beam deflected. An initial "reference" reading of the strain indicator readout will be obtained for each gage with the beam undeflected, and a final reading with the beam deflected. The differences in these two sets of readings will give the strains at the respective gage locations.

3. Referring to Figure 3, connect the lead wires from the gages to the posts on the sides of the "flexor" frame. Referring to Figure 4, with the loading screw clear of the beam, connect one of the two common leads (in the Flexor cable) to the S- binding post of the P-3500 Strain Indicator, and the other common lead to the DI2O post. Connect the independent lead from Gage (1) to the P+ binding post. The gage is used in a "quarter-bridge" arrangement by connecting the lead labeled as 2 to the DI20 post on the P-3500.
4. Using Section 12.9 for guidance, depress the Amp Zero button and balance the amplifier. Then depress the Gage Factor button and set the

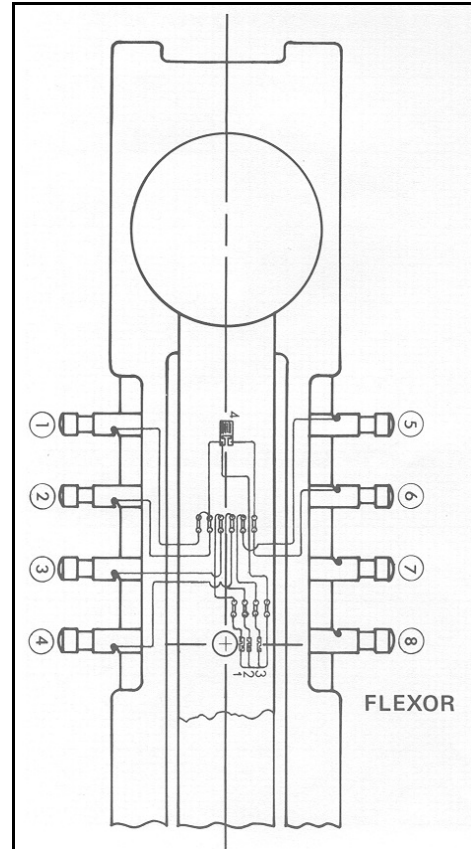


Figure 3. Flexor wiring diagram.

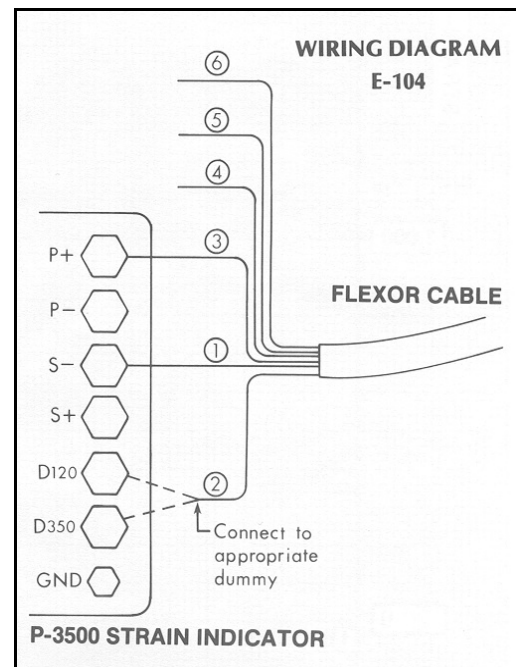


Figure 4. Connections to the P-3500.

gage factor (as displayed in the LCD readout) for Gage (1) to the value listed by the manufacturer on the cantilever beam. Note that this gage factor setting will also be used for Gage (4).

That is, **do not adjust the gage factor setting during the course of the experiment, even though the gage factor for Gage (4) differs from Gages(1), (2), and (3).** A simple correction will be made later to account for the difference in gage factor, if it exists.

Select the X1 MULT position and depress the RUN push button. With the Flexor loading screw still clear of the beam, adjust the balance control of the P-3500 until the LCD digital display indicates exactly 0. Do not adjust the balance control again during the experiment. The initial (zero-beam-deflection) reading for Gage (1) should now be recorded on the worksheet as 0  $\mu\epsilon$ .

5. Turn the indicator off. Disconnect the independent lead of Gage (1) from the P+ binding post, leaving the common leads connected. Connect the independent lead from Gage (2) to the P+ binding post and turn the instrument on. Without adjusting the balance controls, note the reading appearing in the indicator display. This is the initial reading for Gage (2), and should be recorded on the work sheet. Repeat this procedure for Gages (3) and (4), remembering to leave the gage factor the same, and the balance control fixed in its original position at all times.
6. After obtaining and recording the initial reading for Gage (4), add 2000  $\mu\epsilon$  to the value of this initial reading. With Gage (4) connected, deflect the beam, by rotating the loading screw clockwise, until the indicator readout registers the number equal to this sum. The strain at the Gage (4) location (and the nominal strain at the hole centerline) is now 2000  $\mu\epsilon$ , except for a small gage-factor correction which can be made later.
7. Turn off the strain indicator, disconnect the independent Gage (4) lead from the instrument, and reconnect the Gage (3) lead. Turn the instrument on again and note the number registered by the indicator readout. This number is the second reading for Gage (3) and should be recorded in the appropriate location on the work sheet. Repeat this procedure for Gages (2) and (1). While Gage (1) is still connected to the strain indicator, a simple check on system stability can be made. Back the loading screw away until it clears the beam. The indicator display should now read 0  $\mu\epsilon$ , since this was the initial setting for Gage (1). If the indication is more than, say, 10  $\mu\epsilon$  from 0, the source of the error should be located and the experiment repeated.

## 9.5 Laboratory Report

### FIGURES:

Include a sketch of the beam, indicate dimensions, point of load, support, and the discontinuity.

### CALCULATIONS:

1. Calculate the strain in each gage by subtracting the initial strain reading from the final strain reading.
2. Correct the strain indicated by gage 4 (if its gage factor differs from the setting) by using the relationship:

$$\epsilon_{4_{corrected}} = 2000 \frac{S_{g1}}{S_{g4}} .$$

3. Compute the coefficients in Equation (9.3-10).
4. Compute the maximum strain at the edge of the hole,  $\epsilon_{B_{maximum}}$  using Equation (9.3-11).
5. Compute the stress concentration factor,  $K_t$ , by using Equation (9.3-12).
6. Compare the value in Step 5 to that found by using (Figure 2) and the values for  $d/t$  and  $d/b$ .

## WORK SHEET FOR STRESS CONCENTRATION

### BEAM DIMENSIONS:

b = \_\_\_\_\_ inches (width)  
t = \_\_\_\_\_ inches (thickness)  
d = \_\_\_\_\_ inches (diameter of hole)  
x = \_\_\_\_\_ inches (from clamp to centerline of gages 1-3)  
L = \_\_\_\_\_ inches (from clamp to applied load)

### GAGE FACTORS:

$S_{g1}$  = \_\_\_\_\_ [gage factor for Gage (1)]  
 $S_{g2}$  = \_\_\_\_\_ [gage factor for Gage (2)]  
 $S_{g3}$  = \_\_\_\_\_ [gage factor for Gage (3)]  
 $S_{g4}$  = \_\_\_\_\_ [gage factor for Gage (4)]

### STRAIN MEASUREMENTS:

Gage	Initial reading ( $\mu\epsilon$ )	Final reading ( $\mu\epsilon$ )
1		
2		
3		
4		

## CALCULATION SHEET FOR STRESS CONCENTRATION

### 1. INDIVIDUAL STRAINS FOR GAGES:

$$\epsilon = \text{FINAL READING} - \text{INITIAL READING}$$

$$\epsilon_1 = \text{_____} - \text{_____} = \text{_____} \mu\epsilon$$

$$\epsilon_2 = \text{_____} - \text{_____} = \text{_____} \mu\epsilon$$

$$\epsilon_3 = \text{_____} - \text{_____} = \text{_____} \mu\epsilon$$

### 2. CORRECTION OF $\epsilon_4$ FOR GAGE FACTOR:

$$\epsilon_{4_{corrected}} = 2000 \frac{S_{gl}}{S_{g4}} = 2000 \frac{(\text{_____})}{(\text{_____})} = \text{_____} \mu\epsilon$$

### 3. COMPUTATION OF CONSTANT COEFFICIENTS:

$$C = 5.86 (\epsilon_1 - \epsilon_2) - 5.44 (\epsilon_2 - \epsilon_3)$$

$$= 5.86 [(\text{_____} - \text{_____})] - 5.44 [(\text{_____} - \text{_____})] = \text{_____}$$

$$B = 3.49 (\epsilon_1 - \epsilon_2) - 1.20 C$$

$$= 3.49 [(\text{_____} - \text{_____})] - 1.20 \text{_____} = \text{_____}$$

$$A = \epsilon_1 - 0.743 B - 0.552 C$$

$$= \text{_____} - 0.743 \text{_____} - 0.552 \text{_____} C = \text{_____} .$$

### 4. MAXIMUM STRAIN AT EDGE OF HOLE:

$$\epsilon_{B_{maximum}} = A + B + C = \text{_____} + \text{_____} + \text{_____} = \text{_____}$$

5. STRESS CONCENTRATION FACTOR:

$$K_t = \frac{\epsilon_{B_{maximum}}}{\epsilon_{4_{corrected}}} = \frac{(\quad)}{(\quad)} = \underline{\hspace{2cm}}$$

6. THEORETICAL STRESS CONCENTRATION FACTOR:

$$\frac{d}{b} = \frac{(\quad)}{(\quad)} = \underline{\hspace{2cm}}$$

$$\frac{d}{t} = \frac{(\quad)}{(\quad)} = \underline{\hspace{2cm}}$$

$$K_t = \underline{\hspace{2cm}}$$

7. COMPARISON OF EXPERIMENTAL TO THEORETICAL:

$$\frac{K_{t_{theory}} - K_{t_{experiment}}}{K_{t_{theory}}} \times 100 = \underline{\hspace{2cm}} \%$$

## CHAPTER 10 - PRINCIPAL STRESSES AND STRAINS

### 10.1 Objective

The purpose of this experiment is to measure the strains along three different axes surrounding a point on a cantilever beam, calculate the principal strains and then the principal stresses from these strains, and compare the result with the stress calculated from the flexure formula. The experiment should take approximately 30 minutes to run; a maximum group size of 4 people is recommended.



### 10.2 Materials and Equipment

- Cantilever flexure frame (see Section 12.6 for details)
- No. B103 (2024-T6 high-strength aluminum alloy beam); 1/8 x 1 x 12.5 in. (3x25 x320 mm)
- P-3500 strain indicator (see section 12.9 for details)
- Micrometer (see Section 12.1 for details)
- Calipers (see Section 12.2 for details)
- Scale
- Weights and hanger

### 10.3 Background

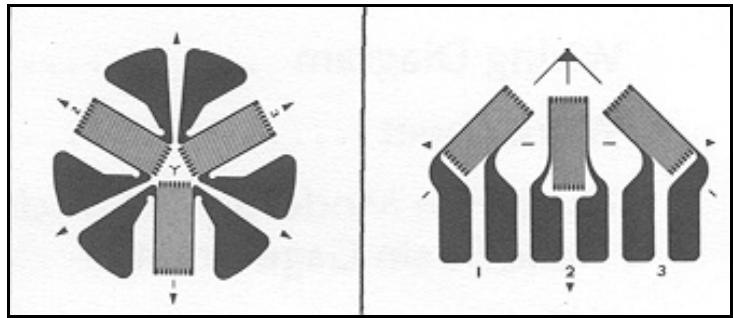
At every point in a solid body there are three mutually perpendicular planes that are called the principal planes. When a Cartesian axes system is assigned, the unit vectors along the normals to these planes are called eigenvectors. By definition, there are no shear stresses on the principal planes, consequently, the normal stresses are aligned with the eigenvectors. The magnitudes of the normal stresses are referred to as eigenvalues. The three principal stresses (eigenvalues) include the maximum and minimum values of the normal stress at the point in question. The eigenvectors and eigenvalues are found by using transformation equations that rely on rotations of the coordinate axes.

When a point is located on a free surface (plane stress), one of the eigenvectors is normal to the surface. The corresponding eigenvalue is equal to zero. The other two eigenvectors (principal stress directions) and eigenvalues (principal stresses) are found by considering a rotation of coordinates around this eigenvector. This can be done analytically by using the transformation equations or graphically by using Mohr's circle.

Since the in-plane principal stresses are always perpendicular to one another, it is sufficient to determine the angle from a reference axis to only one of them. Since the magnitudes of the stresses are also unknown, three independent equations are required to fully establish the stress state. In the case of a linearly elastic, homogeneous, and isotropic material, the directions of principal stress and strain coincide, and three independent strain measurements can be made to determine the two in-plane principal stresses and their directions. In most cases, this is done by measuring the normal strains along three different directions.

The three axes along which strains are measured can be arbitrarily oriented about the point of interest. For computational convenience, however, it is preferable to space the measurement axes apart by submultiples of  $\pi$ , such as  $\pi/3$  ( $60^\circ$ ) or  $\pi/4$  ( $45^\circ$ ). An integral array of strain gages intended for simultaneous multiple strain measurements about a point is known as a "rosette".

Figure 1 shows two of the most common commercially available strain rosettes. The rosette at the left is called a "delta" or equiangular rosette while the rosette at the right is a  $45^\circ$  rectangular rosette. The configurations are analyzed by the manufacturer who provides the governing equations required to extract critical information regarding the strain and the stress.



**Figure 1.** The delta (left) and rectangular (right) rosette.

To perform this analysis, an axis is aligned with one of the gages. The general strain transformation

$$\epsilon_\theta = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \gamma_{xy} \sin 2\theta \quad (10.3-1)$$

is applied to each of the gages with  $\theta$  measured counterclockwise from the x axis. The three equations are manipulated to obtain  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$  in terms of the three normal strain readings taken from the gages ( $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ ). Then, a final transformation is made to obtain the principal values.

For the delta rosette, the principal strains ( $\epsilon_p$  and  $\epsilon_q$ ) and the orientation of the principal axes ( $\theta_p$  and  $\theta_q$ ) are given in terms of the three measured strains as,

$$\epsilon_{p,q} = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} \pm \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} \quad (10.3-2)$$

and

$$\theta_{p,q} = \frac{1}{2} \tan^{-1} \frac{\sqrt{3} (\epsilon_3 - \epsilon_2)}{2 \epsilon_1 - \epsilon_2 - \epsilon_3} . \quad (10.3-3)$$



The rectangular rosette, on the other hand, is characterized by,

$$\epsilon_{p,q} = \frac{\epsilon_1 + \epsilon_3}{2} \pm \frac{1}{\sqrt{2}} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2} \quad (10.3-4)$$

and

$$\theta_{p,q} = \frac{1}{2} \tan^{-1} \frac{2 \epsilon_2 - \epsilon_1 - \epsilon_3}{\epsilon_1 - \epsilon_3} . \quad (10.3-5)$$

The algebraically maximum ( $\epsilon_p$ ) and minimum ( $\epsilon_q$ ) principal strains correspond to the plus and minus alternatives, respectively. As discussed in Section 7.3, even though there is no stress normal to the surface (*i.e.*,  $\sigma_z = 0$ ), the surface deforms in this direction and the corresponding strain ( $\epsilon_z$ ) is

$$\epsilon_z = - \frac{\nu}{(1 - \nu)} [\epsilon_p + \epsilon_q] \quad (10.3-6)$$

where  $\nu$  is the Poisson's ratio.

The information in Equation (10.3-6) can be introduced into the generalized Hooke's Laws to obtain the simplified expressions for the biaxial case at hand. For plane stress, the principal stresses are given by

$$\sigma_p = \frac{E}{1 - \nu^2} ( \epsilon_p + \nu \epsilon_q ) \quad (10.3-7)$$

and

$$\sigma_q = \frac{E}{1 - \nu^2} ( \epsilon_q + \nu \epsilon_p ) \quad (10.3-8)$$

where  $E$  is the elastic modulus.

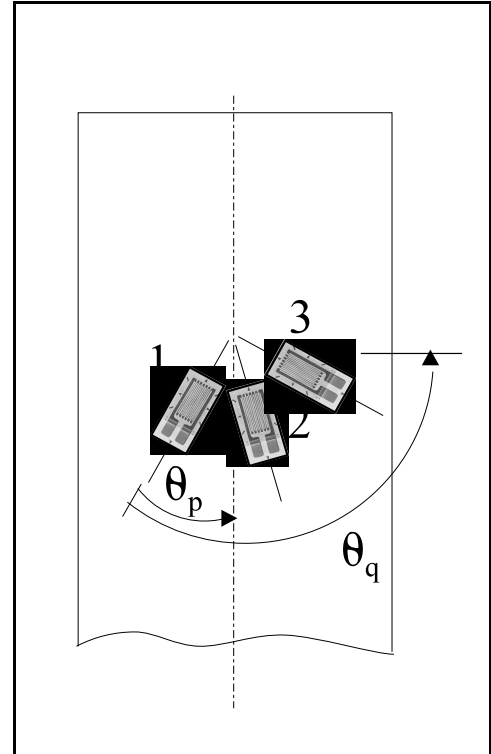
Since the principal directions of stress and strain coincide for the material in question, Equations such as (10.3-3) and (10.3-5) can be used to obtain the principal stress directions.

Our objective is to measure the strains along three different axes surrounding a point on the free surface of a cantilever beam using a rectangular rosette, calculate the principal directions [by using Equation (10.3-5)], strains [from Equation (10.3-4)] and stresses [Equations (10.3-7) and (10.3-8)], and compare these results with those obtained from the flexure formula [Equation (8.3-9)].

Figure 2 shows a schematic of the rectangular rosette mounted on the specimen along with the convention used by the manufacturer while developing Equations (10.3-4) and (10.3-5). Recall that the stress field is uniaxial for points on the surface of the cantilever (other than those near the load or clamped end). The directions of the principal stresses correspond to the longitudinal and lateral directions; the stress along the longitudinal direction are given by the flexure formula while the stress in the transverse direction is equal to zero.

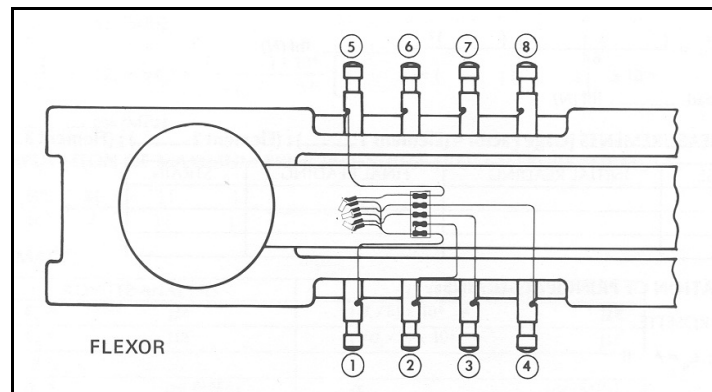
#### 10.4 Procedure

1. Back the calibrated loading screw out of the way, and insert the beam into the Flexor with the rosette on the upper surface. Center the free end of the beam between the sides of the Flexor, and firmly clamp the beam in place with the knurled clamping screw.
2. Measure and record the beam width ( $b$ ), beam thickness ( $t$ ), distance from the centerline of the rosette to the fixed support ( $x$ ), distance between the fixed support and the load ( $L$ ), gage factors for the gages as listed on the beam ( $S_{g1}$ ,  $S_{g2}$ ,  $S_{g3}$ ), and the angles of orientation ( $\theta_p$  and  $\theta_q$ ) of Gage (1) with respect to the longitudinal and transverse axes of the beam, respectively.



**Figure 2.** Rosette on test specimen.

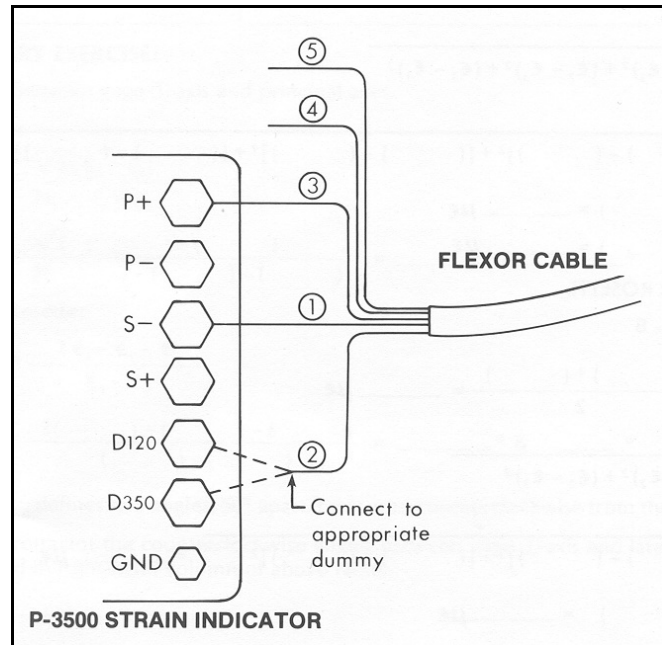
3. Using Equation (6.3-3), determine the load,  $P$ , to be applied for a stress,  $\sigma$ , of 15,000 psi to result at the strain gage rosette. This is  $P_{max}$  (a few pounds).
4. Referring to Figure 3, connect the lead wires from the rosette to the posts on the sides of the "flexor" frame. Referring to Figure 4, connect the appropriate gage leads from the Flexor cable to the S-, P+, and D-120 binding posts of the P-3500 strain indicator so that the strain in Gage (1) will be read. Note: The gage is used in a "quarter-bridge" arrangement by connecting the lead labeled as 2 to the



**Figure 3.** Wiring diagram for Experiment 4.

D120 post on the P-3500.

- Using Section 12.9 for guidance, depress the Amp Zero button and balance the amplifier. Then depress the Gage Factor button and set the gage factor (as displayed in the LCD readout) for Gage (1) to the value listed by the manufacturer on the cantilever beam. Select the X1 MULT position and depress the RUN push button. With the beam unloaded (except by its own weight and the weight of the loading hook), use the balance controls of the P-3500 to achieve a bridge balance (as indicated by a zero in the LCD readout). Do not adjust the balance controls again for the remainder of the experiment. The initial (zero-beam-deflection) reading for Gage (1) should now be recorded on the worksheet as  $0 \mu\epsilon$ .



**Figure 4.** Connections to the P-3500.

- Turn the indicator off. Disconnect the independent lead of Gage (1) from the P+ binding post, leaving the common leads connected. Connect the independent lead from Gage (2) to the P+ binding post and turn the instrument on. Change the gage factor to that of Gage (2) and, without adjusting the balance controls, note the reading on the indicator display. This is the initial reading for Gage (2), and should be recorded on the work sheet. Repeat this procedure for Gage (3), remembering to set the correct gage factor and leaving the balance control fixed in its original position at all times.
- After recording the initial reading for Gage (3), leave the gage connected to the instrument and apply the previously calculated load (or approximately that amount) with weights hanging on the free end of the beam. Record the exact weight on the work sheet and record the indicated strain in the table.
- Leaving the load on the beam, repeat the above procedure for Gages (1) and (2). While the last gage is still connected to the instrument, remove the load from the beam leaving the hanger in position. The strain indicator readout should now indicate the same reading, within a few  $\mu\epsilon$ , as the initial reading for this gage. If the readings are not closely coincident, the source of error should be located and the experiment repeated.

## 10.5 Calculations

1. Obtain the strain in each gage by subtracting the initial reading from the final. Retain the sign of the strain. A negative strain indicates a compressive strain.
2. Substitute these three strains,  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ , into the equations for principal strains,  $\epsilon_{p,q}$ . Correct the minimum principal strain for transverse sensitivity by multiplying it by 1.025. The error in the maximum principal strain is negligible.
3. Calculate Poisson's ratio from the principal strains. Since the stress field is uniaxial with the principal axes along the longitudinal and lateral axes, Poisson's ratio is:

$$\nu = - \left[ \frac{\epsilon_{lateral}}{\epsilon_{longitudinal}} \right] .$$

4. Compute the angles between Gage (1) and the principal axes.
5. Calculate the experimental principal stress by using the Poisson ratio found in Step 3, the principal strains found in Step 2, with  $\epsilon_q$  corrected for transverse sensitivity, and a modulus of elasticity of  $10.4 \times 10^6$  psi and Hooke's law.
6. Compute, by using the flexure formula and the applied load, the theoretical principal stresses at the rosette.
7. Compare values obtained from the flexure formula and measuring the angles to those obtained from strain measurements.

## 10.6 Laboratory Report

### FIGURES:

Include a sketch of the beam indicating dimensions. Indicate location and orientation of strain rosette.

### CALCULATIONS:

1. Provide a summary of flexure formula vs. strain gage values.
2. What are some sources of error? Where would these types of gages be useful?

## WORK SHEET FOR PRINCIPAL STRESSES AND STRAINS

### BEAM DIMENSIONS:

b = \_\_\_\_\_ inches (width)  
t = \_\_\_\_\_ inches (thickness)  
x = \_\_\_\_\_ inches (from clamp to centerline of rosette)  
L = \_\_\_\_\_ inches (from clamp to applied load)

### GAGE FACTORS:

S<sub>g1</sub> = \_\_\_\_\_ [gage factor for Gage (1)]  
S<sub>g2</sub> = \_\_\_\_\_ [gage factor for Gage (2)]  
S<sub>g3</sub> = \_\_\_\_\_ [gage factor for Gage (3)]

### ORIENTATION ANGLES:

θ<sub>p</sub> = \_\_\_\_\_ degrees [angle between the axis of Gage (1) and the longitudinal axis of beam]  
θ<sub>q</sub> = \_\_\_\_\_ degrees [angle between the axis of Gage (1) and the lateral axis of beam]

### MAXIMUM LOAD (for 15,000 psi):

$$P_{\max} = \frac{\sigma b t^2}{6(L-x)} = \frac{(15000 \text{ lb/in}^2) (\text{_____ in.}) (\text{_____ in.})^2}{6 [(\text{_____ in.}) - (\text{_____ in.})]} = \text{_____ lb}$$

$$ACTUAL P = \text{_____ lb}$$

### STRAIN MEASUREMENT:

Gage	Initial reading (μϵ)	Final reading (μϵ)
1		
2		
3		

## CALCULATION SHEET FOR PRINCIPAL STRESSES AND STRAINS

### 1. INDIVIDUAL STRAINS FOR GAGES IN THE ROSETTE:

$$\epsilon = \text{FINAL READING} - \text{INITIAL READING}$$

$$\epsilon_1 = \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \mu\epsilon$$

$$\epsilon_2 = \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \mu\epsilon$$

$$\epsilon_3 = \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \mu\epsilon$$

### 2. PRINCIPAL STRAINS:

$$\epsilon_p = A + B \quad \epsilon_q = (A - B) \cdot C$$

where  $C$  is a correction factor for transverse sensitivity

$$A = \frac{\epsilon_1 + \epsilon_3}{2} = \frac{(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})}{2} = \underline{\hspace{2cm}} \mu\epsilon$$

$$B = \frac{1}{\sqrt{2}} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2}$$

$$= \frac{1}{\sqrt{2}} \sqrt{[(\underline{\hspace{1cm}}) - (\underline{\hspace{1cm}})]^2 + [(\underline{\hspace{1cm}}) - (\underline{\hspace{1cm}})]^2} = \underline{\hspace{2cm}} \mu\epsilon$$

$$\epsilon_p = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \mu\epsilon$$

$$\epsilon_q = (\underline{\hspace{2cm}} - \underline{\hspace{2cm}}) \cdot 1.025 = \underline{\hspace{2cm}} \mu\epsilon$$

### 3. POISSON'S RATIO:

$$\nu = \left| \frac{\epsilon_q}{\epsilon_p} \right| = \left| \frac{(\underline{\hspace{1cm}})}{(\underline{\hspace{1cm}})} \right| = \underline{\hspace{2cm}}$$

4. COMPUTATION OF ANGLES BETWEEN GAGE (1) AND PRINCIPAL AXES:

$$\theta_{p,q} = \frac{1}{2} \tan^{-1} \frac{2 \epsilon_2 - \epsilon_1 - \epsilon_3}{\epsilon_1 - \epsilon_3}$$

$$\theta_{p,q} = \frac{1}{2} \tan^{-1} \frac{2 ( \quad ) - ( \quad ) - ( \quad )}{( \quad ) - ( \quad )}$$

$$= \text{_____}^\circ \quad \text{and} \quad \text{_____}^\circ$$

5. COMPUTATION OF PRINCIPAL STRESSES:

$$\sigma_p = \frac{E}{1 - \nu^2} ( \epsilon_p + \nu \epsilon_q ) = \frac{10.4 \times 10^6}{1 - ( \quad )^2} [( \quad ) + ( \quad ) ( \quad )] \times 10^{-6}$$

$$= \text{_____} \text{ psi}$$

$$\sigma_q = \frac{E}{1 - \nu^2} ( \epsilon_q + \nu \epsilon_p ) = \frac{10.4 \times 10^6}{1 - ( \quad )^2} [( \quad ) + ( \quad ) ( \quad )] \times 10^{-6}$$

$$= \text{_____} \text{ psi}$$

6. CALCULATION OF PRINCIPAL STRESSES FROM THEORY:

$$\sigma_{longitudinal} = \frac{6 P (L - x)}{b t^2} = \frac{6 ( \quad ) [( \quad ) - ( \quad )]}{( \quad ) ( \quad )^2} = \text{_____} \text{ psi}$$

$$\sigma_{lateral} = 0 \text{ psi}$$

## 7. SUMMARY:

Quantity	Rosette	Theory <sup>1</sup>	Errors
$\varepsilon_p$		$\sigma/E =$	
$\varepsilon_q$		$-\nu\sigma/E =$	
$\sigma_p$		$\sigma =$	
$\sigma_q$		0	
$\theta_p$		measured =	
$\theta_q$		measured =	

---

<sup>1</sup> The stress,  $\sigma$ , is along the longitudinal direction.



## CHAPTER 11 - BEAM DEFLECTION TEST

### 11.1 Objective

The purpose of this experiment is to measure the deflection at various points along a simply supported beam and compare these values with theoretical values calculated from the double integration method. It should take approximately 45 minutes to complete the experiment; a maximum group size of 4 people is recommended.

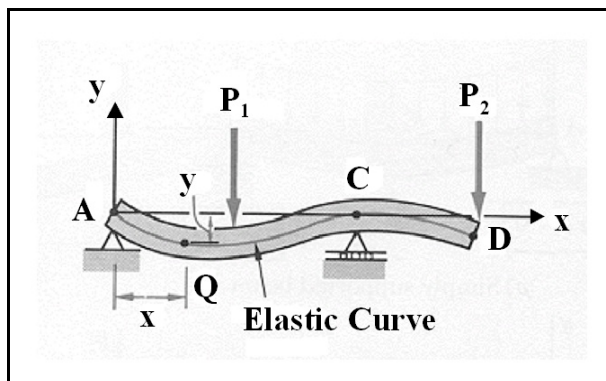


### 11.2 Materials and Equipment

- Frame with Movable Knife Edge Supports (see Section 12.7 for details)
- Test beam(s)
- Micrometer (see Section 12.1 for details)
- Calipers (see Section 12.2 for details)
- Hangers
- Weights

### 11.3 Background

Figure 1 illustrates that when a beam deflects due to lateral loads, the vertical projection of the neutral plane deforms. Points along this plane are said to be on the elastic curve.



**Figure 1.** The elastic curve, figure taken from Beer, Johnston, DeWolf, and Mazurek, “Mechanics of Materials.”

The curvature,  $(1/\rho)$ , at any point on this curve is,

$$\frac{1}{\rho} = \frac{M}{EI} \quad (11.3-1)$$

where  $\rho$  is the radius of curvature,  $E$  is the elastic modulus,  $M$  is the bending moment at the section in question,  $I$  is the centroidal moment of inertia computed around the bending axis. It must be noted that Equation (11.3-1) accounts for the deflection induced by flexure stresses but does not take shearing stresses into account.

Mathematically, the expression for the curvature is,

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2}} \quad (11.3-2)$$

where y is the deflection and x is the position along the longitudinal axis.

For small deflections the slope, dy/dx, is small; therefore,  $(dy/dx)^2 \ll 1$ . Thus, the denominator of the expression for the curvature can be approximated as 1.0 for small deflection problems. In this case, Equation (11.3-2) reduces to

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} \quad (11.3-3)$$

Equating the right hand sides of the expressions contained in Equations (11.3-1) and (11.3-3),

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (11.3-4)$$

where  $M = M(x)$ ; that is, M is a function of position along the span. If the moment expression can be determined in terms of x, then integrating twice with respect to x will yield the expression for deflection, y. The constants of integration are evaluated from the boundary conditions.

The procedure for calculating the deflection of a beam by the double integration method is:

- Solve for the reactions of the beam by applying the two dimensional equilibrium equations to a free body diagram of the beam (or, when distributed loads are involved, a statically equivalent beam).
- Define a coordinate system on the beam, preferably with the origin placed at one of the supports, or, at the beginning of the interval of interest. Note the limits on x for the interval(s) of interest.
- Express the moment over the span as a function of distance by passing sections through arbitrary points. A new section must be considered whenever a discontinuity in shear or moment occurs.
- Apply Equation (11.3-4) in each interval(s). Integrate once to obtain an expression for the

slope. Integrate again for the deflection equation.

- Evaluate the integration constants by using the boundary conditions.<sup>1</sup>
- Check the resulting equations for dimensional homogeneity.

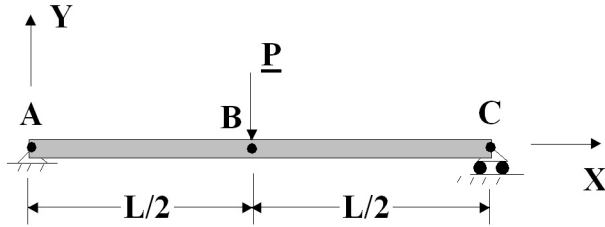
During the analysis, it is often beneficial to construct a shear and a moment diagram. The moment diagram makes it easier to see how this quantity change over the span.

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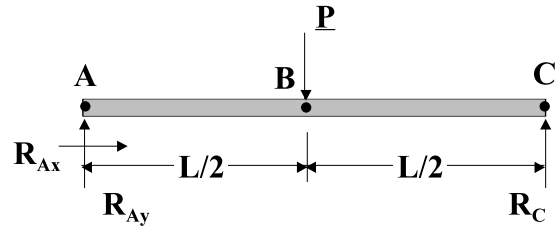
<sup>1</sup> If deflections are desired for spans where the moment expression changes, more than one interval must be used. The same procedure is used but, in addition to boundary conditions, matching conditions are used where the intervals meet. This is accomplished by equating the deflection ( $y$ ) and the slope ( $dy/dx$ ) at the interfaces. The constants of integration are determined from the boundary and matching conditions.

## EXAMPLE NUMBER 1

Figure 2 shows a simply supported beam (AC) with a load applied at the center of the span (B). The free body diagram for the beam is shown in Figure 3.



**Figure 2.** Centrally loaded beam.



**Figure 3.** Free body diagram of beam.

Applying the equilibrium equations to the latter,

$$\begin{aligned}
 \rightarrow F_x = 0 &= R_{Ax} \quad \Rightarrow \quad R_{Ax} = 0 \\
 +\curvearrowright \sum M_A = 0 &= -\frac{P L}{2} + R_C L \quad \Rightarrow \quad R_C = \frac{P}{2} \\
 +\uparrow F_y = 0 &= R_{Ay} - P + R_C \quad \Rightarrow \quad R_{Ay} = \frac{P}{2} .
 \end{aligned}
 \tag{11.3-5}$$

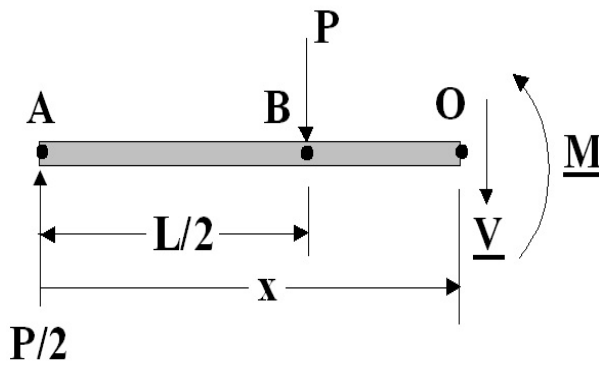
Since there is a discontinuity in load at B, two sections must be considered to obtain the expressions for the shear and moment. Figure 4, for example, shows a section passes through an arbitrary point called O, located somewhere between points A and B. The standard convention is used to denote the shear force,  $V$ , and the bending moment,  $M$ . Applying equilibrium to this section,

$$\begin{aligned}
 +\uparrow F_y = 0 &= -V + \frac{P}{2} \quad \Rightarrow \quad V = \frac{P}{2} \\
 +\curvearrowright \sum M_O = 0 &= M - \frac{P x}{2} \quad \Rightarrow \quad M = \frac{P x}{2} .
 \end{aligned}
 \tag{11.3-6}$$

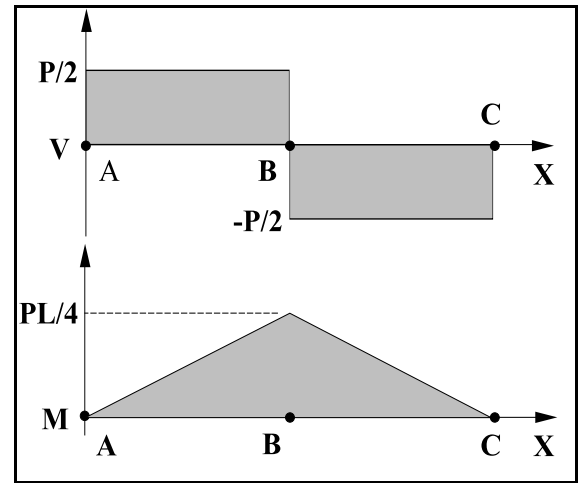
**Figure 4.** A section between A and B.

The expressions in Equation (11.3-6) are valid in the region  $0 \leq x \leq L$ .

Figure 5, on the other hand, shows the section to the right of the applied load.



**Figure 5.** A section between points B and C.



**Figure 6.** Shear and moment diagrams.

Applying equilibrium to this section,

$$\begin{aligned}
 +\uparrow F_y = 0 &= -V + \frac{P}{2} - P \quad \Rightarrow \quad V = -\frac{P}{2} \\
 +\curvearrowright \sum M_O = 0 &= M - \frac{P x}{2} + P \left(x - \frac{L}{2}\right) \quad \Rightarrow \quad M = -\frac{P x}{2} + \frac{P L}{2} .
 \end{aligned}
 \tag{11.3-7}$$

The expressions in Equation (11.3-7) are valid in the region  $L/2 \leq x \leq L$ .

Figure 6 shows the shear and bending moment diagrams obtained by plotting the information in Equations (11.3-6) and (11.3-7) along the span.

Applying Equation (11.3-4) to regions AC and CB,

$$\begin{aligned}
 E I \frac{d^2 y}{dx^2} &= \frac{P x}{2} & 0 \leq x \leq \frac{L}{2} \\
 E I \frac{d^2 y}{dx^2} &= -\frac{P x}{2} + \frac{P L}{2} & \frac{L}{2} \leq x \leq L .
 \end{aligned}
 \tag{11.3-8}$$

Integrating once,

$$\begin{aligned}
 E I \frac{dy}{dx} &= \frac{P x^2}{4} + C_1 & 0 \leq x \leq \frac{L}{2} \\
 E I \frac{dy}{dx} &= -\frac{P x^2}{4} + \frac{P L x}{2} + C_3 & \frac{L}{2} \leq x \leq L .
 \end{aligned}
 \tag{11.3-9}$$

Integrating a second time,

$$E I y = \frac{P x^3}{12} + C_1 x + C_2 \quad 0 \leq x \leq \frac{L}{2} \quad (11.3-10)$$

$$E I y = -\frac{P x^3}{12} + \frac{P L x^2}{4} + C_3 x + C_4 \quad \frac{L}{2} \leq x \leq L .$$

When the boundary conditions ( $x = 0, y = 0$ ) and ( $x = L, y = 0$ ) are applied to the upper and lower expressions in Equation (11.3-10), respectively,

$$C_2 = 0 \quad \text{and} \quad 0 = \frac{P L^3}{6} + C_3 L + C_4 . \quad (11.3-11)$$

The matching conditions require that the expressions for the slope included in Equation (11.3-9) equal one another at the interface where  $x = L/2$ . The same holds true for the deflection expressions given by Equation (11.3-10). The results obtained by satisfying these matching conditions are,

$$C_1 - C_3 = \frac{P L^2}{8} \quad \text{and} \quad C_1 - C_3 = \frac{P L^2}{12} + \frac{2 C_4}{L} . \quad (11.3-12)$$

Solving the expressions included in Equations (11.3-11) and (11.3-12) simultaneously,

$$C_1 = -\frac{P L^2}{16} \quad C_2 = 0 \quad C_3 = -\frac{3}{16} P L^2 \quad C_4 = \frac{P L^3}{48} . \quad (11.3-13)$$

Substituting the values of these constants into Equations (11.3-9) and (11.3-10) yields,

$$E I \frac{dy}{dx} = \frac{P x^2}{4} - \frac{P L^2}{16} \quad 0 \leq x \leq \frac{L}{2} \quad (11.3-14)$$

$$E I \frac{dy}{dx} = -\frac{P x^2}{4} + \frac{P L x}{2} - \frac{3 P L^2}{16} \quad \frac{L}{2} \leq x \leq L .$$

and

$$E I y = \frac{P x^3}{12} - \frac{P L^2 x}{16} \quad 0 \leq x \leq \frac{L}{2} \quad (11.3-15)$$

$$E I y = -\frac{P x^3}{12} + \frac{P L x^2}{4} - \frac{3 P L^2 x}{16} + \frac{P L^3}{48} \quad \frac{L}{2} \leq x \leq L$$

respectively.

## EXAMPLE NUMBER 2

Figure 7 shows a simply supported beam (BC) with overhanging loads positioned at points A and D. The free body diagram for the beam is shown in Figure 8.

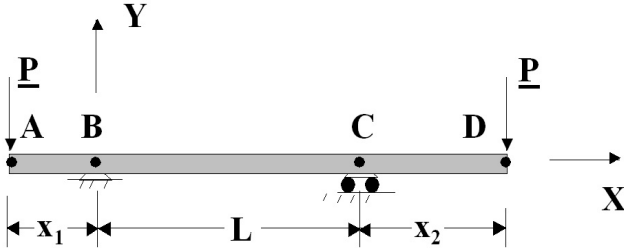


Figure 7. Beam with overhanging loads.

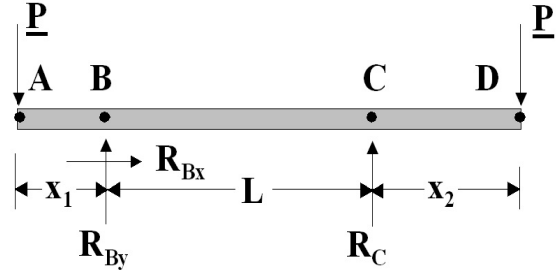


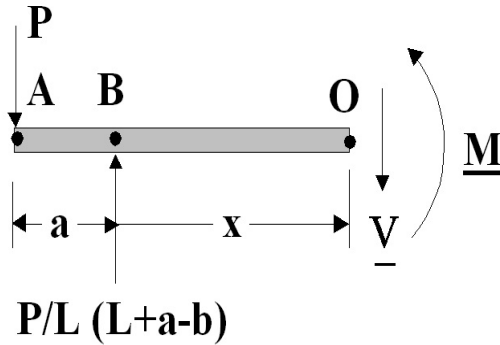
Figure 8. Free body diagram of beam.

Applying the equilibrium equations to the latter,

$$\rightarrow F_x = 0 = R_{Bx} \Rightarrow R_{Bx} = 0$$

$$\uparrow \sum M_B = 0 = P a + R_C L - P (L + b) \Rightarrow R_C = \frac{P}{L} (L + b - a) \quad (11.3-16)$$

$$\uparrow \sum M_C = 0 = -P b - R_{By} L + P (L + a) \Rightarrow R_{By} = \frac{P}{L} (L + a - b) .$$



The origin is selected at point B, since the interval between the supports is of interest. Figure 9, shows the free body diagram of a section passed through an arbitrary point (O) in this interval. The standard convention is used to denote the shear force, V, and the bending moment, M.

Applying equilibrium to this section,

$$+\uparrow F_y = 0 = -V - P + \frac{P}{L} (L + a - b) \Rightarrow V = \frac{P}{L} (a - b)$$

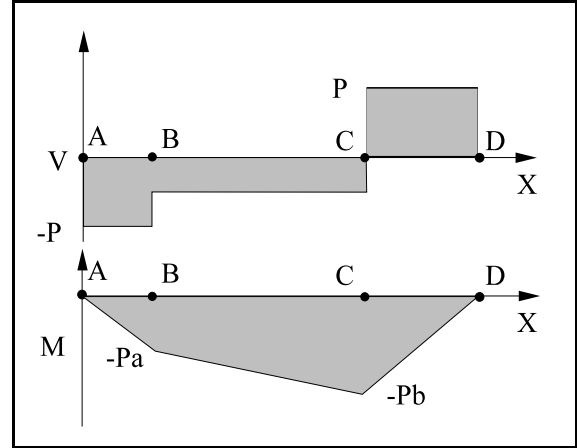
$$\curvearrowright \sum M_O = 0 = M + P (x + a) - \frac{P x}{L} (L + a - b) \quad (11.3-17)$$

$$\Rightarrow M = \frac{P x}{L} (a - b) - P a .$$

The expressions in Equation (11.3-17) are valid in the region  $0 \leq x \leq L$ .

Figure 10 shows the shear and bending moment diagrams obtained by inspection using  $q = -dV/dx$  and  $V = dM/dx$  where  $q$  is the applied load. The assumption has been made that  $a < b$ . The shapes between the supports can be verified by plotting the information obtained from the cut section.

Applying Equation (11.3-4) to region BC,



**Figure 10.** Shear and moment diagrams.

$$E I \frac{d^2 y}{dx^2} = \frac{P x}{L} (a - b) - P a \quad 0 \leq x \leq L \quad (11.3-18)$$

Integrating once,

$$E I \frac{dy}{dx} = \frac{P x^2}{2 L} (a - b) - P a x + C_1 \quad 0 \leq x \leq L \quad (11.3-19)$$

Integrating a second time,

$$E I y = \frac{P x^3}{6 L} (a - b) - \frac{P a x^2}{2} + C_1 x + C_2 \quad 0 \leq x \leq L \quad (11.3-20)$$

Applying the boundary conditions ( $x = 0, y = 0$ ) and ( $x = L, y = 0$ ),

$$C_2 = 0 \quad \text{and} \quad C_1 = \frac{P L}{6} (2 a + b) \quad (11.3-21)$$

Substituting the values of these constants into Equations (11.3-19) and (11.3-20) yields,

$$E I \frac{dy}{dx} = \frac{P x^2}{2 L} (a - b) - P a x + \frac{P L}{6} (2 a + b) \quad 0 \leq x \leq L \quad (11.3-22)$$



and

$$E I y = \frac{P x^3}{6 L} (a - b) - \frac{P a x^2}{2} + \frac{P L}{6} (2 a + b) x \quad 0 \leq x \leq L \quad (11.3-23)$$

respectively.

## 11.4 Procedure

For each beam or loading condition:

1. Set the knife edge supports at the determined positions.
2. Measure the width and thickness of the beam. Record these, along with the type of material on the data sheet.
3. Position the beam on the supports.
4. Place the loading hook(s) and hanger(s) at the point(s) of load(s).
5. Place dial indicators at selected points along the beam.
6. Record the positions of the indicators and the load(s).
7. Be certain that the indicators will give a positive reading for the loading scheme. Check that the gages have sufficient travel. If not, loosen the screw on the back and move the gage, being certain to center the plunger before tightening the screw.
8. Zero the gages. Tap them lightly to be certain they are settled.
9. Apply 5 loads to the beam. Record the load and deflection at each gage.

## 11.5 Laboratory Report

### FIGURES:

Include sketches of the test set up with placement of loads and indicators. Note beam cross section shape and dimensions.

### CALCULATIONS:

For every specimen tested, use the equations derived by the double integration method to calculate the theoretical deflections under each of the dial gages. Calculate the percent error by comparing the experimentally measured deflections to those predicted by the theory. **As explained in Section 11.6, the theoretical results must be computer generated.**

### GRAPH:

For every specimen tested, plot the load versus deflection for each dial gage.

## WORK SHEET FOR BEAM DEFLECTION

### CENTRALLY LOADED BEAM

CRITICAL DIMENSIONS:

L = \_\_\_\_\_ inches (test length)

w = \_\_\_\_\_ inches (width)

t = \_\_\_\_\_ inches (thickness)

POSITIONS OF DIAL GAGES:

$x_1$  = \_\_\_\_\_ inches (position of gage number 1)

$x_2$  = \_\_\_\_\_ inches (position of gage number 2)

$x_3$  = \_\_\_\_\_ inches (position of gage number 3)

$x_4$  = \_\_\_\_\_ inches (position of gage number 4)

MATERIAL: \_\_\_\_\_

DEFLECTION DATA:

STEP	LOAD	GAGE 1	GAGE 2	GAGE 3	GAGE 4
1					
2					
3					
4					
5					

## WORK SHEET FOR BEAM DEFLECTION

### OVERHANGING LOADS

CRITICAL DIMENSIONS:

L = \_\_\_\_\_ inches (test length)  
w = \_\_\_\_\_ inches (width)  
t = \_\_\_\_\_ inches (thickness)  
a = \_\_\_\_\_ inches (distance from left support to left overhanging load)  
b = \_\_\_\_\_ inches (distance from right support to right overhanging load)

POSITIONS OF DIAL GAGES:

$x_1$  = \_\_\_\_\_ inches (position of gage number 1)  
 $x_2$  = \_\_\_\_\_ inches (position of gage number 2)  
 $x_3$  = \_\_\_\_\_ inches (position of gage number 3)

MATERIAL: \_\_\_\_\_

DEFLECTION DATA:

STEP	LOAD	GAGE 1	GAGE 2	GAGE 3
1				
2				
3				
4				
5				

## 11.6 Computer Requirement

As mentioned in Section 11.5, the theoretical solution for this experiment must be computer generated. Students may use a simple computer code written in any language, a spread sheet such as Excel, or, any other mathematical solver. The method must be specified in the report and a table containing the theoretical values must be included.

To this end, the theoretical deflections for each load and position are given for the center loading in Equation (11.3-15) as

$$E I y = \frac{P x^3}{12} - \frac{P L^2 x}{16} \quad 0 \leq x \leq \frac{L}{2} \quad (11.3-15)$$

$$E I y = -\frac{P x^3}{12} + \frac{P L x^2}{4} - \frac{3 P L^2 x}{16} + \frac{P L^3}{48} \quad \frac{L}{2} \leq x \leq L$$

and for the overhung loading in Equation (11.3-23) as

$$E I y = \frac{P x^3}{6 L} (a - b) - \frac{P a x^2}{2} + \frac{P L}{6} (2 a + b) x \quad 0 \leq x \leq L \quad (11.3-23)$$

In the above equations, the only unknown is the theoretical deflection “y”. All other parameters are known.

The requirement is to obtain a theoretical deflection for each load, at every location where an experimental deflection measurement was recorded. As an example, consider the following Microsoft Excel spreadsheet generated to solve the first expression in Equation (11.3-15):

	A	B	C	D	E	F
1	E (psi)	I (in <sup>4</sup> )	P (lb)	x (in)	L (in)	y (in)
2	30000000	0.0013	2	4	20	-0.0048547

The procedure used to generate the table is:

In row 1, label each cell by the parameter associated with that cell.

In row 2, provide the actual values of all known values. Note that in the above example random values have been chosen for E, I, P, x, and L.

The equation to solve for ‘y’ must be typed in row 2, column F. The following is exactly what would have to be typed there:

$$=((c2/12)*d2^3-(c2*e2^2/16)*d2)/(a2*b2)$$

In the above formula, 'c2' refers to the number in column C, row 2. The '^' symbol is used for exponents; the '/' and '\*' symbols are used to divide and multiply, respectively.

Once the above formula has been typed in column F, row 2, press enter or return. The value for 'y' should appear in the cell.

Data can be plotted by highlighting the appropriate cells and using the plot icon. A scatter plot is suggested, since the maximum and minimum values of the axes can be user selected.

## CHAPTER 12 - INSTRUMENTATION AND EQUIPMENT

### 12.1 Micrometers

The micrometers supplied in the laboratory are accurate to 0.001 of an inch or 0.01 mm. However, micrometer accuracy can be affected by temperature changes, shocks during transportation, etc., and should be checked and adjusted before use. Figures 1 and 2 illustrate the salient features of the device.

#### 12.1.1 Reading a micrometer:

In particular, Figure 1 shows a micrometer with thousandth (0.001)" graduations. Each number graduation on the barrel equals 0.100" (one hundred thousandths). Each line graduation on the barrel, between the number graduations, equals 0.025" (twenty-five thousandths). Each line graduation on the thimble equals 0.001" (one thousandth).

Figure 2, on the other hand, shows a micrometer with metric graduations. Each number graduation on the barrel equals 1.0 mm. Each line graduation on the barrel, between the number graduations, equals 0.5 mm. Each line graduation on the thimble equals 0.01 mm.

Figure 3 shows how to hold a micrometer while testing. To read the instrument, it is necessary to take the highest number graduation visible on barrel and add the number of line graduations visible. Add to this total the number of the thimble line graduation which coincides with the long horizontal line of the barrel. Thus, the readings on the micrometers shown in Figures 1 and 2 are 0.394 in. and 8.44 mm, respectively.

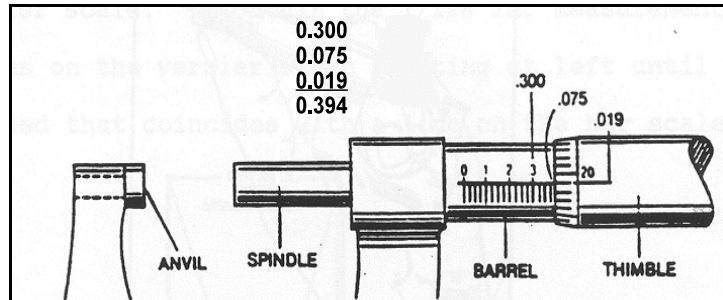


Figure 1. A micrometer graduated in inches.

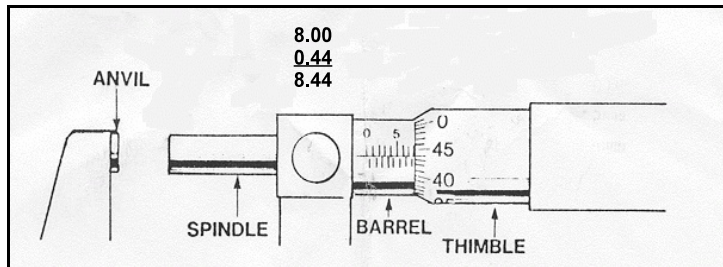


Figure 2. A micrometer graduated in millimeters.

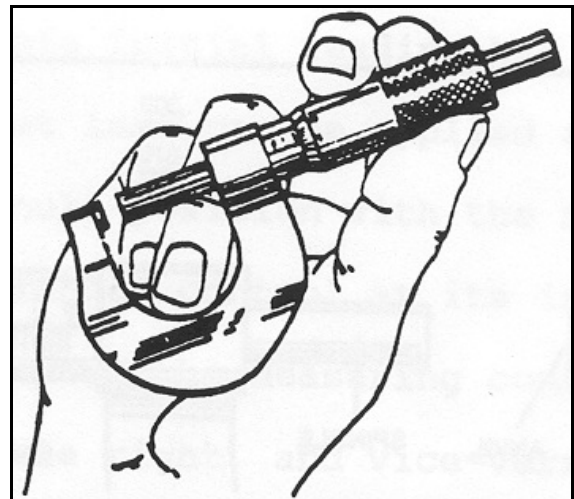


Figure 3. Making the measurement.

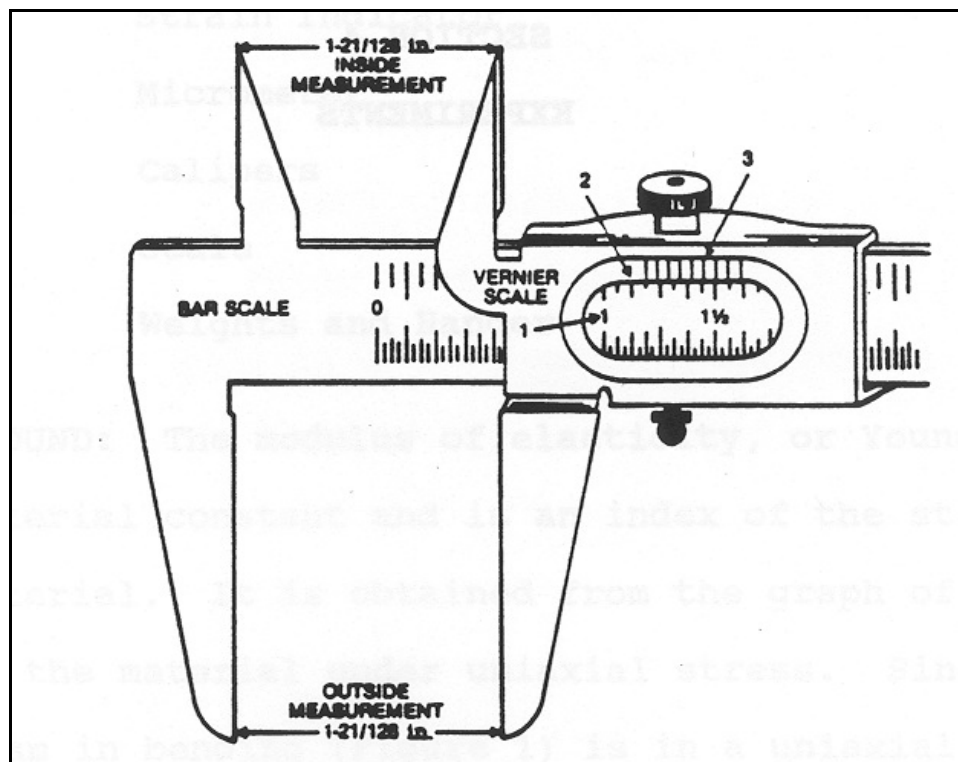
### 12.1.2 Making adjustments:

To adjust for wear: loosen the screw in the end of the thimble. Close the spindle onto the anvil with the same pressure used in measuring. With a hex wrench, loosen the setscrew in the thimble and move the thimble so that the 0 mark is lined up with the 0 mark on barrel. Tighten the screw with the wrench. Retighten the screw in the end of the spindle.

To ensure accurate setting: make sure the spindle does not turn while moving the thimble or tightening the setscrew.

## 12.2 Calipers

Figure 4 illustrates the salient features of a vernier calipers. Each inch on the bar scale is divided into 1/16ths, and each of the 8 spaces on the vernier scale corresponds to 1/128 in. (or 1/2 of 1/64 in.).



**Figure 4.** A vernier calipers.

To read the instrument, add the measurements as follows: The full inches on the bar scale plus the 16ths on the bar scale to the first line on the vernier scale. To obtain the 1/128 in. measurement, count the spaces on the vernier scale starting at left until a line is reached that coincides with a line



on the bar scale.

To calculate the measurement illustrated in Figure 4, for example:

Add:

1 full inch on the bar scale	=	1 in.
2/16 inch on the bar scale	=	16/128 in.
5/128 inch on the vernier scale	=	5/128 in.
<hr/>		
Total measurement	=	1 21/128 in.

### 12.3 Tensile Testing Machine

The tensile testing machine consists of an electro-mechanical test system that applies uniaxial loading in a uniform manner to test specimens. It is general purpose in its capabilities and applications. The system performs load versus elongation (stress versus strain) tests which involve controlling forces from a few ounces to several-thousand pounds, gripping specimens ranging from delicate fibers to high strength metals or composites, and measuring the resulting forces (stresses) and deformations (strains). As illustrated in Figure 5, measurement of the stresses and strains is accomplished by the use of highly sensitive load and strain transducers that create an electrical signal that is proportional the applied stress or strain. This electrical signal is measured, digitized and then processed for display, analysis and report of stress, strain and other computed material characteristics.



**Figure 5.** Tensile testing machine.

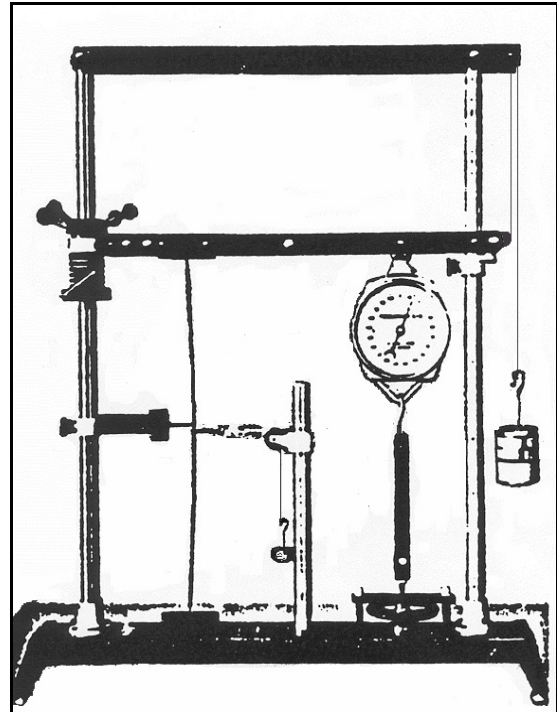
### 12.4 Column Buckling Machine

The column buckling machine enables a series of tests to be carried out to investigate the validity of the Euler formula for three different boundary conditions using columns ranging in length from 15" (38 cm) to 30" (76 cm). To enable deflections to be accurately determined, 0.75" (19.1 mm) x 0.125" (3.2 mm) rectangular section columns are used for all the experiments, thus ensuring that deflection occurs in a predetermined plane.

A slight pre-load may be applied horizontally to the center of the column to enable the point of

buckle to be accurately determined and to ensure that the direction of the buckle is opposite to the measuring dial gage or vernier scale. Referring to Figure 6, the positions of the pulley used to apply the pre-load and the dial gage used to measure deflection have been interchanged on the machines located in the laboratory. The dial gage was mounted on the center post so that the graduated scale located there can be used to accurately position the gage at the center of the column.

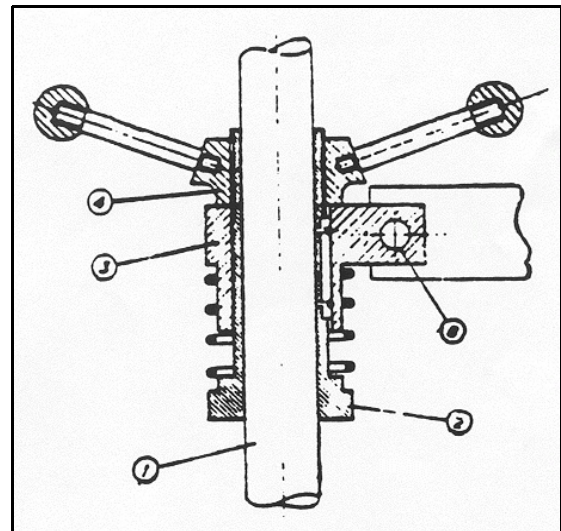
The testing machine consists of a 4" (10.2 cm) x 1.5" (3.8 cm) rectangular tubular steel base carrying two 1.25" (3.2 cm) diameter vertical steel posts connected at their top by two steel tie bars. A unique feature of the machine is the adjustable loading beam pivot located on the left-hand vertical post and illustrated in detail in Figure 7. The threaded split steel sleeve (2) may be clamped in any position on the vertical post (1) to suit the length of the column under test and is set so that the spirit level on the horizontal loading beam indicates that the loading beam is dead level.



**Figure 6.** Column buckling machine.

As the right-hand end of the loading beam falls during loading, the pivot (8) at the left-hand end may be lowered by gently turning the capstan nut (4) which forces the bronze pivot casting (3) down the threaded steel sleeve (2). The loading beam is thereby restored to a dead level position and, by careful adjustment of the capstan nut (4) during loading of the column, it is easily possible to ensure that a true axial load is applied to the column throughout the test.

The loading beam, which is pivoted in ball bearings at its left-hand end, carries a spring balance to which is connected the hand wheel operated loading screw attached to the machine base plate. An extension of the loading beam is supported by an adjustable weight, the suspension cord passing over a ball bearing pulley fixed between the two top tie bars. This adjustable weight counterbalances the weight of the loading beam, the spring balance and the connecting links which connect the loading screw to the spring balance.



**Figure 7.** Adjustable loading beam pivot.

Sets of connecting links are provided with the machine to correspond with the various beam lengths and each set is marked with its respective beam length. The ratio between the distance from the loading beam ball bearing pivot to the column center line and the ball bearing pivot to the loading screw center -- that is the vertical center line of the spring balance -- is 4 to 1. A load on the spring balance of 50 lb (223 N) therefore represents a 200 lb (890 N) load on the column. Note especially, that this ratio has been taken into account in calibrating the spring balance dial which records the true axial load on the column.

The machine was delivered with screw vice fittings on the loading beam and in the machine base for testing columns under fixed end conditions at both ends. Alternative hardened steel blocks with suitable V grooves were supplied for test under pin end conditions. These blocks are easily interchangeable with the screw vice blocks so that the following conditions are readily available:

- Both ends of the column pin jointed.
- Both ends of the column rigidly fixed.
- One end of the column rigidly fixed while the other end is pin jointed.

For the first condition, the columns have 60° angle machined on each end, the locating V blocks having 90° angles. For the second condition, the columns have square ends and, for the last condition, the columns have one end 60° angle and one end square.

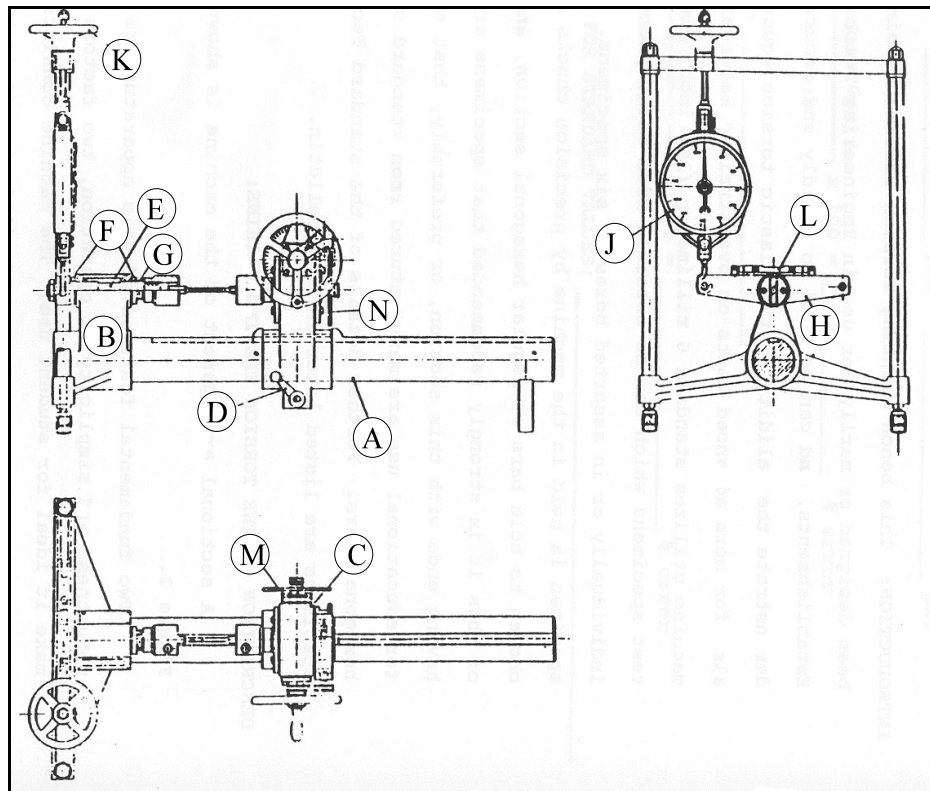
As the critical load for a 15" (38 cm) column with fixed ends is so much greater than the critical load for a 30" (76 cm) column with pin jointed ends, two alternative spring balances are supplied, one reading to a maximum column load of 800 lb (3560 N) and the other to a maximum column load of 80 lb (356 N).

## **12.5 Torsion Testing Machine**

The bench mounted torsion testing machine can be used to rapidly and accurately demonstrate the validity of the elastic torsion equation and, in more advanced tests, on over-strained materials. The machine utilizes standard 0.25 in. (6 mm) diameter cylindrical test specimens with hexagonal ends. Each specimen is held in the machine by precision chucks specially chosen to hold the hexagonal section.

A schematic of the torsion machine is shown in Figure 8. The machine consists of a rigid base (A), formed from a 3 in. (76 mm) diameter ground steel bar, carrying a fixed headstock (B) at its end, and manually operated straining head (C) which is freely moveable along its length. The straining head is basically a worm reduction gear box of 60:1 ratio, which is mounted on a cast iron bracket, and arranged to slide on a keyway along the full length of the base. A clamp (D) is provided to lock the straining head in any position along the base according to the length of specimen under test. A hardened rod ground steel mainshaft (E) through the headstock is of substantial proportions and is mounted in needle roller bearings. It is also free to move longitudinally through the bearings to

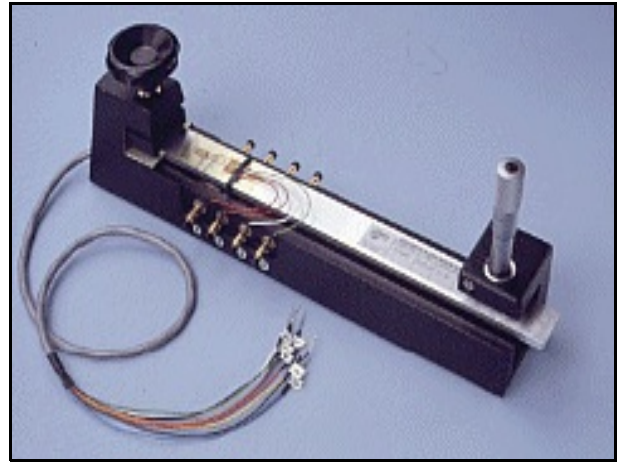
allow for the slight change in length which is normally experienced by the specimen when subjected to a torsional load. Thrust bearings (F) are placed on the shaft at each end of the headstock and the whole shaft assembly spring loaded by the light compression spring (G). Onto the free end of this shaft is fitted a balanced torque arm (H) of 5 in. (125 mm) radius, and the torque, applied to the specimen in either direction, can be measured by a circular spring balance (J) supported from the frame over the headstock. The spring balance dial is calibrated directly in torque units of in-lb. A screw leveling device, operated by the horizontal handwheel (K) on the top of the spring balance frame, returns the torque arm to the horizontal position. A spirit level (L) fixed onto the arm (H) is used in conjunction with the handwheel, to level the unit. Onto the other end of the mainshaft is keyed a precision two jaw chuck. An identical chuck is also fitted to the output shaft of the straining head. The specimen is firmly held between these chucks during the test. The handwheel on the input shaft of the straining head is used to strain the specimen. Straining of the specimen can be applied in either the clockwise or anticlockwise direction, and is measured on the straining head by the two protractor scales (M) and (N), or by a revolution counter directly coupled through a toothed belt to the input shaft. The protractor scale (M), on the input shaft, is graduated from  $0^{\circ}$  to  $6^{\circ}$  in both directions and is subdivided into  $0.5^{\circ}$  and  $0.1^{\circ}$  intervals for determining fine angular displacements of the specimen. The scale (N), on the output shaft, is generated from  $0^{\circ}$  to  $360^{\circ}$  in both directions and subdivided into  $10^{\circ}$  and  $1^{\circ}$  intervals for coarse angular displacement measurements. Both protractors are free to rotate on their appropriate shafts and can be locked in any position on the shaft by knurled thumb nuts. The revolution counter simply records revolutions of the input shaft and therefore each revolution corresponds to  $6^{\circ}$  movement of the output shaft.



**Figure 8.** The torsion testing machine.

## 12.6 Flexor Testing Fame

The Flexor, cantilever flexure frame, is a simple, versatile fixture for stressing beams. As shown in Figure 9, it consists of a rigid cast aluminum frame, with provision for clamping a cantilever beam in position at the one end and for applying fixed displacements to the free end of the beam. Clamping of the beam is accomplished with a captive clamping pad which provides determinate end-fixity and prevents damaging the beam with the steel clamping screw. The loading screw incorporated in a precision micrometer terminates in a polished ball end for applying loads to the beam assembly.



**Figure 9.** The Flexor.

The Flexor is equipped with an integral, eight conductor cable, pre-wired to the eight binding posts, and protected with a strain-relief bushing. Individual conductors are coded to their binding posts by number. To use the unit the test beam is inserted full-depth into the clamping end. Making certain that the free end of the beam is centered between the side rails of the Flexor, the beam is firmly clamped in place. The beam then can be deflected either with the calibrated loading screw or by using dead weights. The small depression in the beam is provided for hanging weights with the loading hook.

When using the calibrated loading screw, the zero-adjust feature can be used to set the dial reading to zero after the ball end of the screw is firmly in contact with, and slightly deflecting the beam. (Because of the linear behavior of the beam over the elastic range of deflections employed in the Flexor, any slightly deflected position can be treated as the zero position, and the strain and deflection measured with respect to this initial condition.)

The micrometer-type loading screw carries an attached barrel graduated in 0.001-in. (0.254 mm) increments of screw travel. On an associated stationary sleeve, an index line for reading the barrel indication is provided along with reference marks denoting 0.025 in. (0.635 mm) and 0.1 in. (2.54 mm) increments of screw travel. With the small spanner wrench, the angular position of the stationary sleeve can be rotated to place the index line under the zero on the barrel scale for the zero-deflection condition.

Strain gage connections can be easily made with the Flexor. The Flexor is equipped with eight push-pin binding posts. This permits simultaneous connection of up to six gage grids in three-wire circuits (with a common pair of leads to all of the gages). Connection of strain gages to the binding posts can be made by short, six-inch (150-mm) leads. The corresponding leads from the integral three-foot (0.9 m) instrument cable are then connected to the strain indicator as appropriate.



A choice of procedures is available for performing experiments on the Flexor when a single static strain indicator must be used with more than one gage installation. The experiment usually will require obtaining strain readings for all gages at two or more deflection conditions of the beam. These measurements can be made by the following procedure:

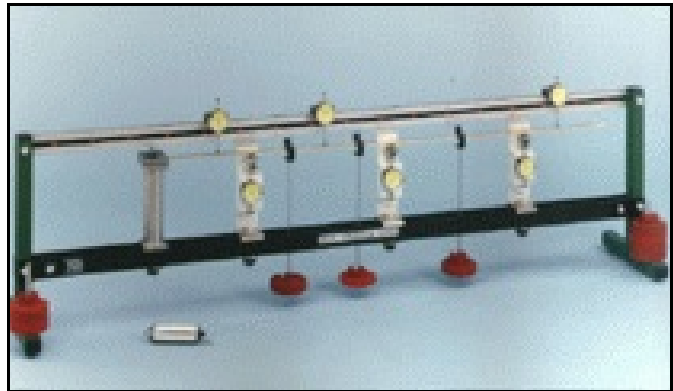
- Establish the first beam deflection condition
- Successively connect the strain gages to the strain indicator and record initial strain readings for all gages
- Establish the second beam deflection condition
- Successively connect the strain gages to the strain indicator and record final strain readings for all gages
- Obtain the strains from the differences in readings for initial and final conditions

An alternative is to take strain readings for both beam conditions for each gage individually and repeat the reading process from gage to gage. With care in reading and setting the calibrated loading screw, or in applying external loads, either method will produce equally accurate data.

A better method, however, is to use a switch-and-balance unit, which permits the pre-connection and independent balancing of up to ten strain gage circuits. A single switch on the unit then can be used to connect each gage circuit to the Strain Indicator under each deflection condition.

## 12.7 Beam Loading Apparatus

The beam loading apparatus shown in Figure 10 provides facilities for supporting beams on simple, built in and sinking supports; applying point loads, and measuring support reactions and beam deflections. It can be used for an almost limitless number of experiments ranging from determination of the elastic modulus for beams of different materials, through to studies of continuous beams with any loading.

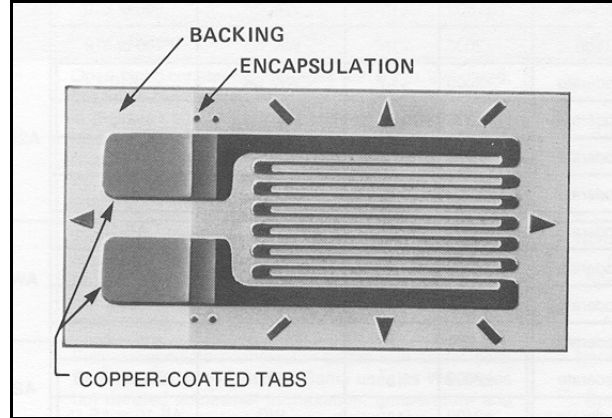


**Figure 10.** Beam loading apparatus.

The main frame of the apparatus consists of an upper cross member carrying graduated scales and two lower members bolted to tee-legs to form a rigid assembly. The cantilever support consists of a rigid pillar with a sturdy clamping arrangement to hold the beams when built-in end conditions are required. Weight hangers and a set of cast iron weights are supplied for applying static loads. All beam deflections are measured by dial gages mounted on carriers which slide along the upper cross member. The dial gage carriers and weight hangers are all fitted with cursors which register on the scale located on the upper cross member, thus ensuring easy, accurate positioning.

## 12.8 Electrical Resistance Strain Gages

Several of the experiments to be conducted utilize cantilever beams that have been equipped with electrical resistance strain gages. As illustrated in Figure 11, each strain gage consists of a metal foil alloy etched into a grid pattern. The gage is bonded to the test beam so that the foil experiences the same normal strain ( $\Delta L/L$ ) as the beam does.



**Figure 11.** An electrical resistance strain gage.

The resistance of the gage,  $R$ , is measured in Ohms ( $\Omega$ ) and may be expressed as

$$R = \frac{\rho L}{A} \quad (12.8-1)$$

where  $\rho$  is the resistivity [measured in  $\Omega$ -in. ( $\Omega$ -m)],  $L$  is the length [in. (m)], and  $A$  is the cross-sectional area [ $\text{in.}^2$  ( $\text{m}^2$ )]

When the strain parallel to the grid is positive, the length of the foil increases while its cross-sectional area decreases. Assuming that the resistivity remains constant, Equation (12.8-1) shows that the resistance of the gage will increase. When a constant current is passed through the gage, this change in resistance,  $\Delta R$ , produces a change in voltage (*i.e.*, Ohm's law;  $V = IR$ ). The voltage change is converted to strain by using a strain indicator.

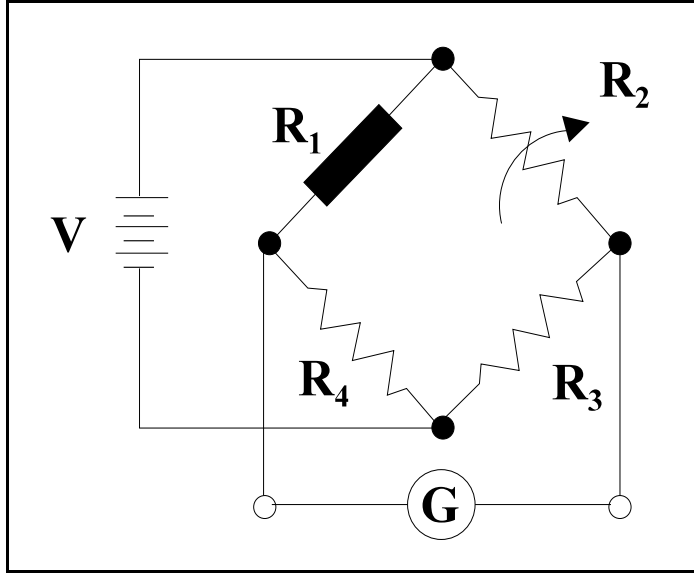
The manufacturer of the gage provides a gage factor,  $S_g$ , so that the change in resistance can be correlated with the strain.  $S_g$  is a dimensionless quantity typically having a magnitude of 2.0. The gage factor equation is given by,

$$S_g = \frac{\Delta R/R}{\epsilon} \quad (12.8-2)$$

where  $R$  is the resistance and  $\epsilon$  is the strain. Rearranging terms,

$$\epsilon = \frac{\Delta R}{R S_g} \quad (12.8-3)$$

Typically,  $R = 120$  ohms and  $S_g = 2.0$ ; strain ranges from  $\epsilon_{\max} = 10,000 \mu \text{ in/in}$  ( $0.01 \text{ in/in}$ ) to  $\epsilon_{\min} = 5 \mu \text{ in/in}$  ( $.000005 \text{ in/in}$ ). Substitution of these values into Equation (12.8-2) shows that the resistance changes to be measured range from 0.012 to 2.4 ohms. The most practical means of accurately measuring these relatively small resistance changes is by using a Wheatstone bridge. The bridge circuit, shown in Figure 12, has an input voltage source,  $V$ , four resistors,  $R$ , and a galvanometer,  $G$ .



**Figure 12.** A Wheatstone Bridge.

In most cases,  $R_1$  is replaced with an active strain gage;  $R_2$  is a variable resistor used to balance, or zero, the bridge; and,  $R_3$  and  $R_4$  remain constant.

The galvanometer is balanced and displays a zero reading when

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \quad (12.8-4)$$

When the gage in  $R_1$  is strained, the bridge becomes unbalanced and

$$\frac{R_1 + \Delta R}{R_4} \neq \frac{R_2}{R_3} \quad (12.8-5)$$

However, the bridge can be rebalanced by adjusting  $R_2$  through a  $\Delta R_m$  such that

$$\frac{R_1 + \Delta R}{R_4} = \frac{R_2 + \Delta R_m}{R_3} \quad (12.8-6)$$

If  $R_1 = R_2 = R_3 = R_4$ , the circuit is balanced when  $\Delta R = \Delta R_m$ . In this case,

$$\varepsilon = \frac{\Delta R_m}{R S_g} \quad (12.8-7)$$

$\Delta R_m$  is measured by using a commercially available instrument called a strain indicator. The input must include specifying the gage factor and the resistance of the gage. Detailed operating instructions for this unit are included in Section 12.9 of this manual.

Since a strain gage measures the average strain over its length, it is usually desirable to make this dimension as small as possible. Figure 3 illustrates that the average of any nonuniform strain distribution is always less than the maximum. Consequently, a strain gage that is noticeably larger than the maximum strain region will indicate a strain magnitude that is too low.

In order to produce a small gage length and achieve the necessary resistance for the Wheatstone Bridge circuit, the manufacturer is forced to etch the foil into loops. The ends of each loop (see

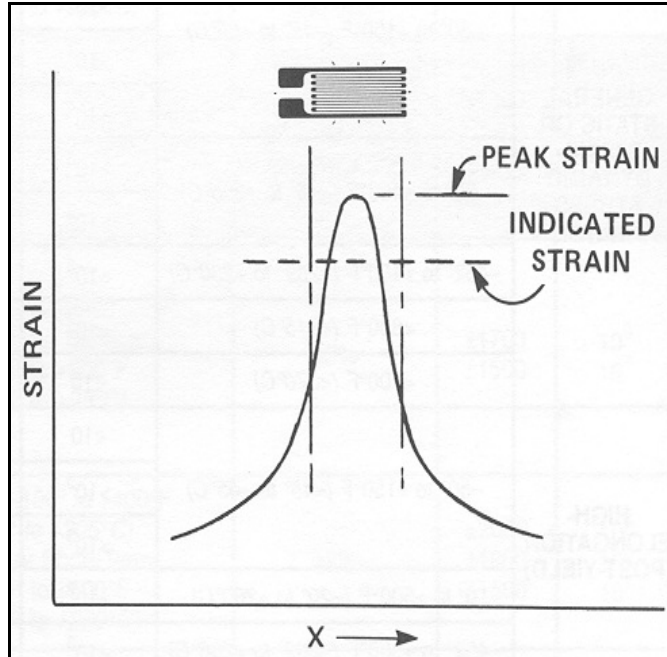


Figure 11) result in a sensitivity to transverse strain.

This effect can be eliminated by introducing a correction factor found based on the transverse sensitivity factor,  $K_t$ , and the ratio of transverse to axial strain.  $K_t$  is provided when needed, and can be used with the charts provided in the manual to obtain the true strain readings.

## 12.9 P-3500 Strain Indicator

The P-3500 is a portable, battery-powered precision instrument that can be used to monitor strain in a surface mounted electrical resistance strain gage. As illustrated in Figure 14, the unit features an LED output. The push-button controls illustrated in Figure 15 provide an easy-to-follow, logical sequence of setup and operational steps.



**Figure 13.** Strain is averaged over the gage length.

### 12.9.1 Connect Strain Gage

Resistive strain gages are normally connected at the binding posts located on the right of the front panel. These binding posts are color-coded in accordance with conventional practice, and are clearly labeled. Input connections for full-, half-, and quarter-bridge configurations are shown on the inside cover of the instrument.

Connect the strain gage leadwires to the binding posts. Select the desired bridge configuration using the BRIDGE push button. Set the MULT push button to X1 position.

### 12.9.2 Initially adjust using AMP ZERO - Orange

Depress the AMP ZERO push button. Allow the instrument to warm up for at least two minutes. Rotate the AMP ZERO finger-tip control for a reading of  $\pm 0000$ . [To save time, the instrument may be left in the



**Figure 14.** P-3500 strain indicator.

AMP ZERO position while the gage(s) is being connected.]

### 12.9.3 Set the GAGE FACTOR - Orange

Set the GAGE FACTOR switch to the desired gage factor range and depress the GAGE FACTOR push button. The gage factor will be displayed on the readout. Rotate the GAGE FACTOR potentiometer for the exact desired gage factor, and lock the control in place. The knob utilizes a lever which must be rotated clockwise to lock it in place. The knob can be unlocked simply by rotating the lever back to the counterclockwise stop.

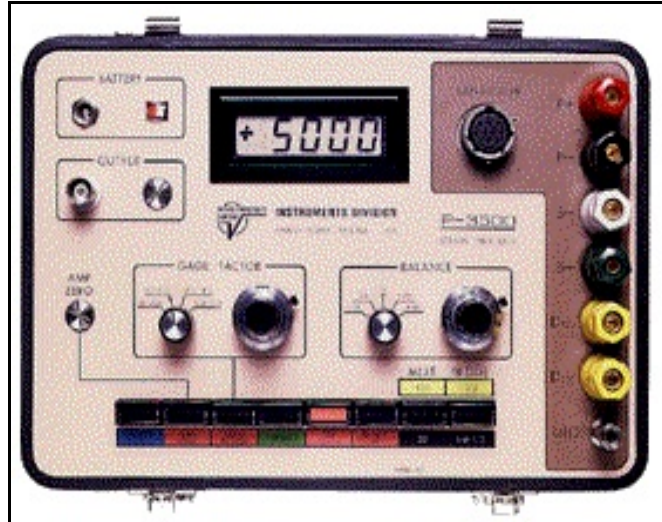


Figure 15. P-3500 push-button control panel.

### 12.9.4 Select RUN - Green

Select the X1 or X10 MULT position as required and depress the RUN push button. In this position all internal circuitry is configured to make an actual strain measurement. Set the BALANCE range switch and rotate the BALANCE potentiometer to obtain a reading of  $\pm 0000$  with no load on the test structure, and lock the control in place. The knob utilizes a lever which must be rotated clockwise to lock it in place. The knob can be unlocked simply by rotating the lever back to the counterclockwise stop.

Note: A reading of  $\pm 0000$  with flashing colons indicates an off-scale condition that is usually caused by improper input wiring or a defective strain gage installation.

The test structure may now be loaded and the reading recorded.

## 12.10 Two-Wire and Three-Wire Strain Gage Circuits

All commercial static strain indicators employ some form of the Wheatstone bridge circuit to detect the resistance change in the gage with strain. When a single active gage is connected to the Wheatstone bridge with only two wires, as illustrated in Figure 16, both of the wires must be in the same leg of the bridge circuit with the gage. One of the effects of this arrangement is that

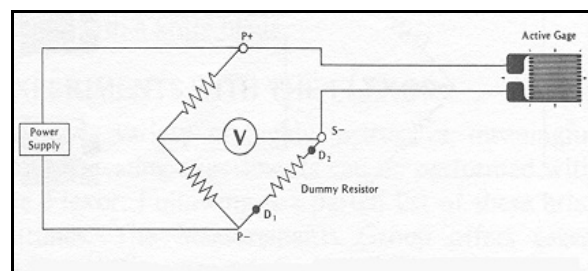
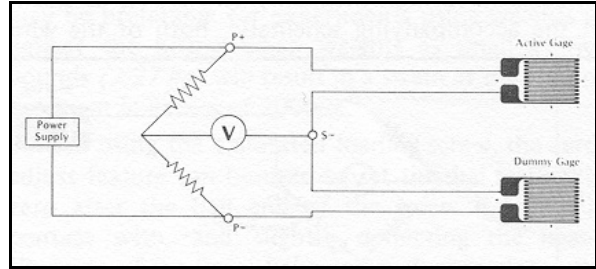


Figure 16. Two-wire circuit for single active gage (quarter bridge).

temperature induced resistance changes in the lead wires are manifested as apparent strain by the strain indicator. The errors due to lead wire resistance changes in single-gage installations with two-wire circuits can be minimized by minimizing the total lead wire resistance; that is, by using short lead wires of the largest practicable cross-section.

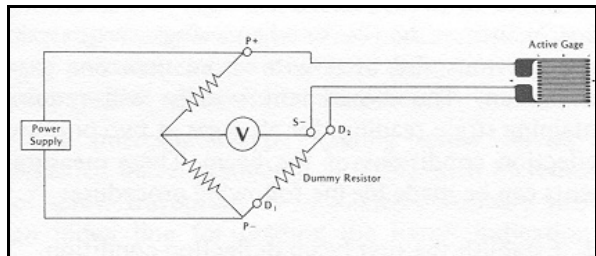
When two matched gages are connected as adjacent legs of the bridge circuit (with the same length lead wires, maintained at the same temperature), the temperature effects cancel since they are the same in each leg, and like resistance changes in adjacent legs of the bridge circuit are self -nullifying. The two-wire circuit for connecting two gages is shown in Figure 17.



**Figure 17.** Two-wire circuit for two gages (half bridge).

Lead wire effects can be virtually eliminated in single active gage installations by use of the "three- wire" circuit. In this case, a third lead, representing the center point connection of the bridge circuit, is brought out to one of the gage terminals. Resistance changes in the bridge center point lead do not affect bridge balance. For this method of lead wire compensation to be effective, the two lead wires in the adjacent bridge arms should be the same length, and should be maintained at the same temperature. The three-wire circuit is the standard method of connection for a single active temperature-compensated strain gage in a quarter-bridge arrangement.

Three-wire circuits can be used with the Flexor by employing three cable leads and three binding posts per gage. When the strain gage on the beam is wired for three-wire connection, the three leads are connected to three binding posts, and the three corresponding cable leads are connected to the strain indicator. This approach is illustrated in Figure 18.



**Figure 18.** Three-wire circuit for single active gage (quarter bridge).

If the strain gage installation is wired with only two leads, a three-wire circuit can still be used between the indicator and the Flexor. Three of the Flexor cable leads and binding posts are used as before. A jumper connection is placed between two of the binding posts. One of these two binding posts is then connected to one of the gage leads, and the third to the other gage lead. This procedure provides lead wire temperature compensation for all but the leads from the gage terminals to the Flexor binding posts.

With three-element rosettes or other multiple-gage installations requiring lead wire compensation, a single pair of cable leads and Flexor binding posts can be used as the common compensating circuitry for all gages. An additional lead for each gage is required to complete the circuit. By this

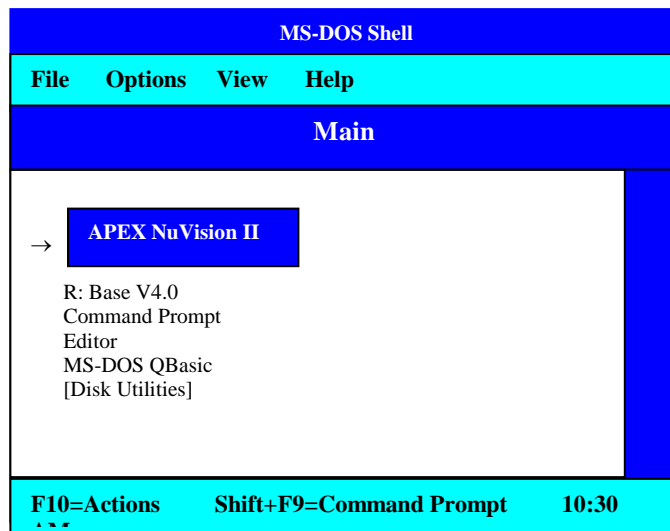
method, a total of six gages can be connected for sequential measurements with a single-channel strain indicator.

Contact resistance at mechanical connections within the Wheatstone bridge circuit can lead to errors in the measurement of strain. Connections should be snugly made. Following bridge balance, a "wiggle" test should be made on wires leading to mechanical connections. No change in balance should occur if good connections have been made.

If necessary, contact surfaces may be cleaned of oils with a low residue solvent such as isopropyl alcohol. If long periods of disuse have caused contact surfaces to tarnish, clean them by scraping lightly with a knife blade.

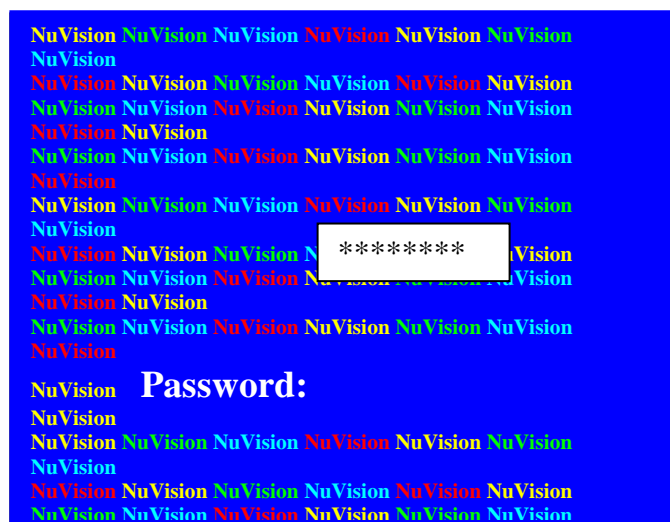
## APPENDIX I - TENSION TEST PROCEDURE

1. Turn on the power surge protector. Turn on the computer, monitor, and printer. The tensile testing machine does not have an on/off control; the machine is controlled by the computer.
2. The following screen will appear on the computer monitor:



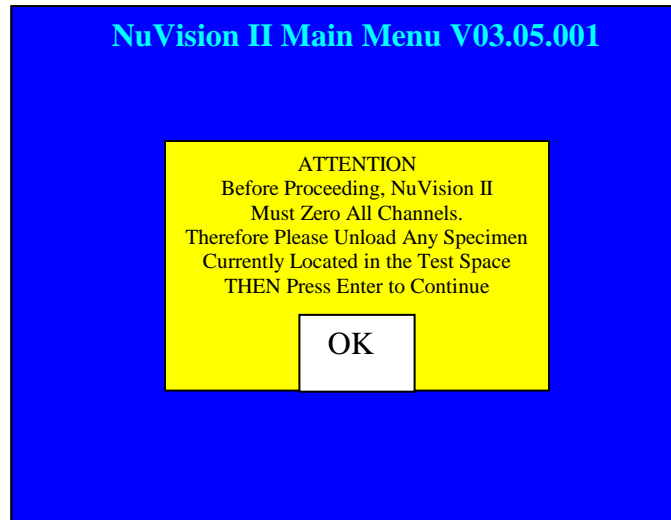
**APEX NuVision II** should be highlighted. If not , use the up and down arrow keys to highlight **APEX NuVision II**. Then press [ENTER].

3. The following screen will appear on the monitor:



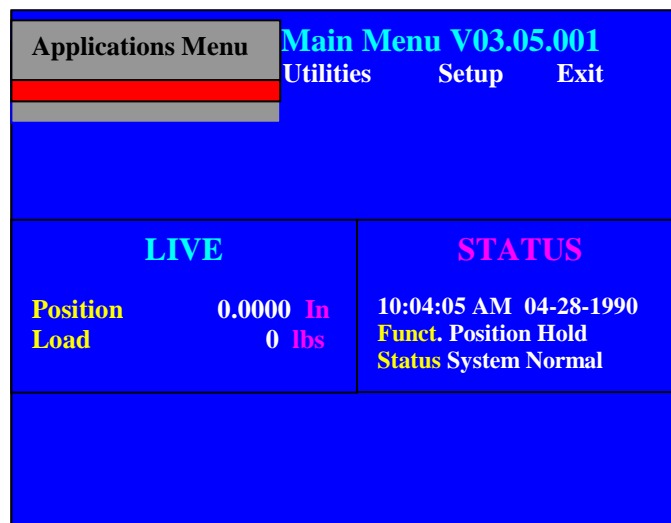
Enter your password. The password has been previously assigned to the laboratory instructor. If a password is not known, the instructor should contact the MAE office for instructions on how to obtain one.

4. After entering the password, the following screen will appear on the monitor:



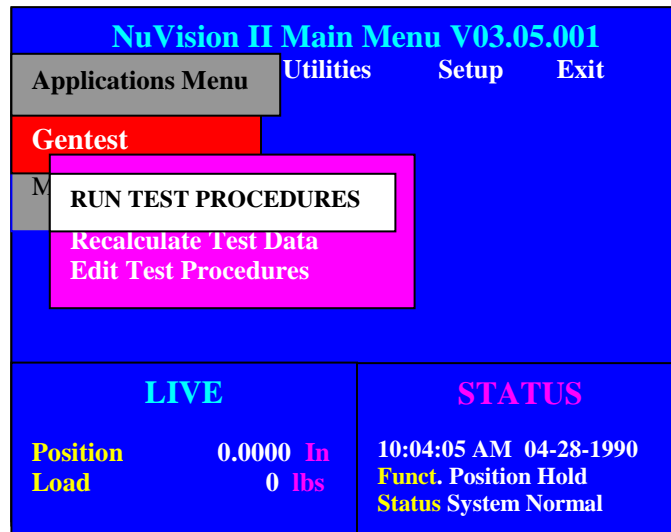
Be sure to remove any specimen located in the test space. Then press [ENTER].

5. The following screen will appear on the monitor:



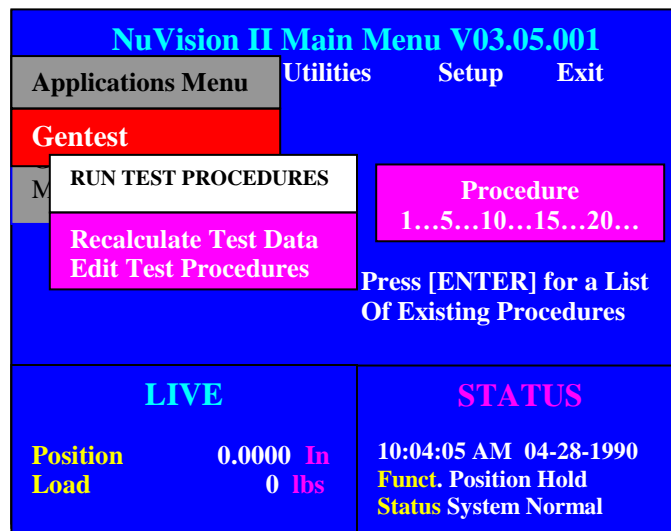
Gentest should be highlighted. If not, use the arrow keys to highlight Gentest. Then press [ENTER].

6. The following screen will appear on the monitor:



Run Test Procedures will be highlighted. If not, use the arrow keys to highlight Run Test Procedures. Then press [ENTER].

7. The following screen will appear on the monitor:



Press [ENTER] for a list of existing procedures.

8. The following screen will appear on the monitor:

NuVision II Main Menu V03.05.001			
Applications Menu		Utilities	Procedures
Gentest		<Type Procedure Name>	
M		<Enter Part Number>	
RUN TEST PROCEDURES		ChemLand	
Recalculate Test Data		Epoxy Samples	
Edit Test Procedures		MAE 294 ALUMINUM	
		MAE 294 BRASS	
		MAE 294 STEEL	
		MAE 370 ALUMINUM	
		Sla-model	
		Test 1	
LIVE		STATUS	
Position	0.0000 In	10:04:05 AM 04-28-1990	
Load	0 lbs	Funct. Position Hold	
		Status System Normal	

<Type Procedure Name> will be highlighted. Use the down arrow key to instead highlight MAE 370 ALUMINUM. Then press [ENTER].

9. The following screen will appear on the monitor:

NuVision II Main Menu V03.05.001			
Applications Menu		Utilities	Gentest MAE 370 ALUMINUM
Gentest		Press 'Esc' to Select A Different Procedure	
M		Press Any Other Key To Run a Test With this Procedure	
RUN TEST PROCEDURES			
Recalculate Test Data			
Edit Test Procedures			
LIVE		Transducers	
Position	0.0000 In	Channel A – Pos. Encoder	
Load	0 lbs	Channel B – None	
		Channel 1 – 10 K Load Cell	
		Channel 2 – None	
		Channel 3 – None	
		Channel 4 – None	
		Channel 5 – None	
		Channel 6 – None	
		Channel 7 – None	
		Channel 8 – None	

The default test procedure appears on the screen. Press [ENTER] to run a test with this procedure.

10. The following screen will appear on the monitor:



Custom Questions	
WEARING SAFETY GLASSES <span>YES</span>	
<b>Before Test Dimensions</b> <div> <div>Width</div> <div>In</div> <div>Thickness</div> <div>Height</div> </div>	<b>STATUS</b> 10:23:34 AM 04-28-1990 Gentest MAE 370 ALUMINUM Funct. Position Hold Status System Normal Test Operator: Ed Hopper Auto Zero On Sample Break 20 % At End of Test Return to Start Start Position ? Mark Position ? Maximum Datasets 270079
<b>Live</b> <div> <div>Position</div> <div>Strain</div> <div>1/I</div> <div>Load</div> </div>	<div> <div>0.00000 In</div> <div>0.00000</div> <div>0</div> </div>

The **Before Test Dimensions** section of the screen will be highlighted. These are the only numbers that need to be changed. The arrow keys are used to move the cursor up and down among the width, thickness, and height indicators.

Measure the exact width, thickness and height of the test specimen. If, for example, the width indicated on screen is different than the measured width of the test specimen, then change the indicated screen value as follows:

- use the up or down arrow key to highlight the width value
- use the right arrow key to position the cursor on the smallest decimal value of the number indicated on screen for the width
- use the backspace key to delete the value
- type in the correct measured width of test specimen.

Repeat for the thickness and height of the test specimen.

11. After entering the correct **Before Test Dimensions** of the specimen, the specimen can be loaded into the SATEC tensile testing machine as follows:

- Use the blue up and yellow down buttons on the tensile testing machine to raise or lower the upper grip to a height above the lower grip of approximately 1" less than the measured height of the test specimen. For example, if the measured height of the test specimen is 6" then align the upper grip a distance of 5" above the lower grip.
- Insert test specimen approximately 1/2" into the upper grip of the tensile testing machine and tighten by rotating the grip handle clockwise. Be sure the test specimen is aligned vertically.
- Insert test specimen approximately 1/2" into the lower grip and tighten by rotating the grip handle counter clockwise. Again, check for vertical alignment of the specimen.

12. After inserting the specimen into the test machine a pre-load of approximately 100 lbs. should be applied to the test specimen. The amount of pre-load will be indicated on the computer screen in the lower left hand corner under the **Live** readings indicated by **Load** in **lbs**. The test specimen should be pre-loaded as follows:
  - Use the up button on the tensile testing machine to *slowly* raise the upper grip in order to apply a tensile load on the test specimen. The amount of load can be simultaneously viewed on the computer screen by the **Load** reading in **lbs**. The pre-load should be at approximately 100 lbs. ***Note:** If this step is being repeated due to insufficient pre-load add an extra 50 Lbs. to the previous pre-load amount.*
13. Once the specimen has been pre-loaded the channels must be zeroed. To accomplish this press the [TAB] key twice and the following screen will appear on the monitor:

<b>Start</b>	Change	Channel	Machine	Exit
<b>Custom Questions</b> <b>WEARING SAFETY GLASSES</b>				<b>YES</b>
<b>Before Test Dimensions</b>		<b>STATUS</b>		
Width		10:23:34 AM 04-28-1990		
In		Gentest		
Thickness		MAE 370 ALUMINUM		
In		Funct. Position Hold		
<b>Live</b> <b>Position</b> 0.45600 In <b>Strain</b> 0.02400 <b>I/I</b> <b>Load</b> 100 lbs		Status System Normal		
		Test Operator: Ed Hopper		
		Auto Zero On		
		Sample Break 20 %		
		At End of Test Return to Start		
		Start Position ?		
		Mark Position ?		
		Maximum Datasets 270079		

**Start** at the top left-hand corner of the screen will be highlighted. Use the right arrow key to highlight **Channel** instead. Press [ENTER].

14. The following screen will appear on the monitor:

Start	Change	Channel	Machine	Exit
<b>Cus</b> <b>WEARING SAFE</b>		<div style="border: 1px solid black; padding: 5px;"> <b>Zero A Channel</b>  Zero All Channels &lt;F2&gt;  Reset A Channel  Reset All Channels </div>		
<b>Before Test Dimensions</b> Width <input type="text"/> In <input type="text"/> Thickness <input type="text"/> In <input type="text"/>		<b>STATUS</b> 10:23:34 AM 04-28-1990 Gentest MAE 370 ALUMINUM <b>Funct.</b> Position Hold <b>Status</b> System Normal <b>Test Operator:</b> Ed Hopper Auto Zero On Sample Break 20 % At End of Test Return to Start Start Position ? Mark Position ? Maximum Datasets 270079		
<b>Live</b> <b>Position</b> 0.45600 In <b>Strain</b> 0.02400 <b>I/I</b> <b>Load</b> 100 Lbs				

**Zero A Channel** will be highlighted. Use the down arrow key to highlight **Zero All Channels <F2>**. Then press [ENTER].

15. The following screen will appear on the monitor:

Start	Change	Channel	Machine	Exit
<b>Cus</b> <b>WEARING SAFE</b>		<div style="border: 1px solid black; padding: 5px;"> <b>Zero A Channel</b>  Zero All Channels&lt;F2&gt;  Reset A Channel  Reset All Channels </div>		
<b>Before Test Dimensions</b> Width <input type="text" value="1"/> In <input type="text"/> Thickness <input type="text" value="6"/> In <input type="text"/>		<b>STATUS</b> 10:23:34 AM 04-28-1990 Gentest MAE 370 ALUMINUM <b>Funct.</b> Position Hold <b>Status</b> System Normal <b>Test Operator:</b> Ed Hopper Auto Zero On Sample Break 20 % At End of Test Return to Start Start Position ? Mark Position ? Maximum Datasets 270079		
<b>Live</b> <b>Position</b> 0.00000 In <b>Strain</b> 0.00000 <b>I/I</b> <b>Load</b> 0 Lbs				

Notice that the **Live** readings have been zeroed. Use the left arrow key to highlight **Start** instead of **Channel** at the top of the screen (**do not** press the [ENTER] key after highlighting **Start**). The following screen will appear on the monitor:

Start		Change	Channel	Machine	Exit
Test Procedure <F11>		Questions			
WEARING SAFETY GLASSES		YES			
Before Test Dimensions			STATUS		
Width		1	10:23:34 AM 04-28-1990		
In			Gentest		
Thickness		6	MAE 370 ALUMINUM		
			Funct. Position Hold		
			Status System Normal		
			Test Operator: Ed Hopper		
			Auto Zero On		
			Sample Break 20 %		
			At End of Test Return to Start		
			Start Position ?		
			Mark Position ?		
			Maximum Datasets 270079		
Live					
Position	0.00000	In			
Strain	0.00000				
I/I					
Load	0	Lbs			

**Test Procedure <F11>** will be highlighted. When ready to begin the tensile test press [ENTER]. If at any time after starting the test there is a need to abort the test press [TAB].

*Note: if the specimen was not inserted correctly or the pre-load was insufficient the test will automatically terminate, the following screen will appear on the monitor and the user should skip to step #26:*

Machine		Pick Points	Calculate
Live		STATUS	
Position	In	10:59:54 AM 04-28-1990	
Strain	I/I	Test Terminated- Break Detected	
Load	Lbs	Funct. Position Hold	
Stress	PSI	Status System Normal	

- If the specimen was inserted correctly and enough pre-load was applied the test should run properly and the following screen will appear on the monitor with the values constantly changing as the specimen is being pulled in tension:

Peaks		<div>Stress v/s Strain Curve</div> <div>(PSI)</div> <div>(I/I)</div>
Position	In	
Strain	I/I	
Load	Lbs	
Stress	PSI	
Valleys		
Position		
In		
Strain		
I/I		
Load		
Lbs		
Stress		
PSI		
Rates		Status
Position		
In		
Strain		
I/I		
Load		
Lbs		
Stress		
PSI		
Live		
Position	0.00000 In	
Strain	0.00000 I/I	
Load	0	
Lbs		
Stress	0 PSI	

Press the space bar to switch between the above screen and a full screen representation of the Stress v/s Strain curve. When the specimen breaks remove the two remaining pieces from the tensile testing machine.

18. After the specimen breaks the screen represented on the monitor will be the following:

Machine		Pick Points	Calculate
Live		STATUS	
Position	In	10:59:54 AM 04-28-1990	
Strain	I/I	Test Terminated- Break Detected	
Load	Lbs	Funct. Position Hold	
Stress	PSI	Status System Normal	

Machine will be highlighted. Use the right arrow key to highlight Calculate. Then press [ENTER].

19. The following screen will appear on the monitor:

<b>Machine</b>		<b>Pick Points</b>	<b>Calculate</b>
<b>And Display Results &lt;ESC&gt;</b>			
<b>Live</b>		<b>STATUS</b>	
<b>Position</b>	<b>In</b>	10:59:54 AM 04-28-1990	
<b>Strain</b>	<b>I/I</b>	Test Terminated- Break Detected	
<b>Load</b>	<b>Lbs</b>	<b>Funct.</b> Position Hold	
<b>Stress</b>	<b>PSI</b>	<b>Status</b> System Normal	

And Display Results <ESC> will be highlighted. Press [ENTER].

20. The following screen will appear on the monitor:

<b>Output</b>	<b>Change</b>	<b>Machine</b>	<b>View</b>	<b>Store</b>	<b>Repeat</b>
<b>WEARING SAFETY GLASSES YES</b>					
Test Gentest					
Procedure MAE 370 ALUMINUM					
Test Date 04-28-1990			Tested by Ed Hopper		
Test Time 10:50:33 AM			Test Counter 00000200		
Elapsed Time 00:02:48			Datasets 4219		
Test Peak 91550 PSI			Area 0.09375 In <sup>2</sup>		
Val@ Break 7396 Lbs			Val@ Break 0.2339 I/I		
Mod-Tangnt 1225039			EUL YIELD PSI		
Off-Yld-Std PSI			Energy-Brk 9596.9 In-Lbs		
<b>Live</b>			<b>STATUS</b>		
<b>Position</b>	<b>#</b>	11:03:27 AM 04-28-1990			
<b>In</b>	<b>#</b>	Test Terminated- Break Detected			
<b>Strain</b>	<b>#</b>	Results & Datapoints Stored			
<b>I/I</b>		<b>Funct.</b> Position Hold			
		<b>Status</b> System Normal			

Output will be highlighted in the upper left-hand corner of the screen. Press [ENTER].

21. The following screen will appear on the monitor:

Output	Change	Machine	View	Store	Repeat
Results & Graph to Printer					
Results to Printer Graph to Printer Graph to Pen Plotter					
Elapsed Time 00:02:48 Test Peak 91550 PSI Val@ Break 7396 Lbs Mod-Tangnt 1225039 Off-Yld-Std PSI			Tested by Ed Hopper Test Counter 00000200 Datasets 4219 Area 0.09375 In <sup>2</sup> Val@ Break 0.2339 I/I EUL YIELD PSI Energy-Brk 9596.9 In-Lbs		
Position Live # In # Strain # I/I ..			STATUS 11:03:27 AM 04-28-1990 Test Terminated- Break Detected Results & Datapoints Stored Funct. Position Hold Status System Normal		

**Results and Graph to Printer** will be highlighted. Verify that the printer is turned on and that sufficient paper is present within printer. Then press [ENTER]. While the results and graph are printing the screen will display the full Stress v/s Strain curve and finally, once again, the screen represented above.

22. To shutdown the computer use the arrow key to highlight **Exit** in the upper right-hand corner of the screen. Press [ENTER] and the following screen will appear:

Output	Change	Machine	View	Store	Repeat	EXIT
WEARING SAFETY GLASSES Y						To Main Menu<ESC>
Test Gentest Procedure MAE 370 ALUMINUM Test Date 04-28-1990 Test Time 10:50:33 AM Elapsed Time 00:02:48 Test Peak 91550 PSI Val@ Break 7396 Lbs Mod-Tangnt 1225039 Off-Yld-Std PSI						To Procedure Editor To Recalc To Specimen ID Editor
Val@ Break 0.2339 I/I EUL YIELD PSI Energy-Brk 9596.9 In-Lbs						
Position Live # In # Strain # I/I ..			STATUS 11:03:27 AM 04-28-1990 Test Terminated- Break Detected Results & Datapoints Stored Funct. Position Hold Status System Normal			

**To Main Menu <ESC>** will be highlighted. Then press [ENTER].

23. The following screen will appear on the monitor:

NuVision II Main Menu V03.05.001			
Applications Menu	Utilities	Setup	Exit
<b>Gentest</b>			
Maintain Test			
<b>LIVE</b>		<b>STATUS</b>	
Position	0.0000 In	10:04:05 AM 04-28-1990	
Load	0 lbs	Funct. Position Hold	
		Status System Normal	

**Gentest** is highlighted. Use the arrow key to highlight **Exit** instead. Then press [ENTER].

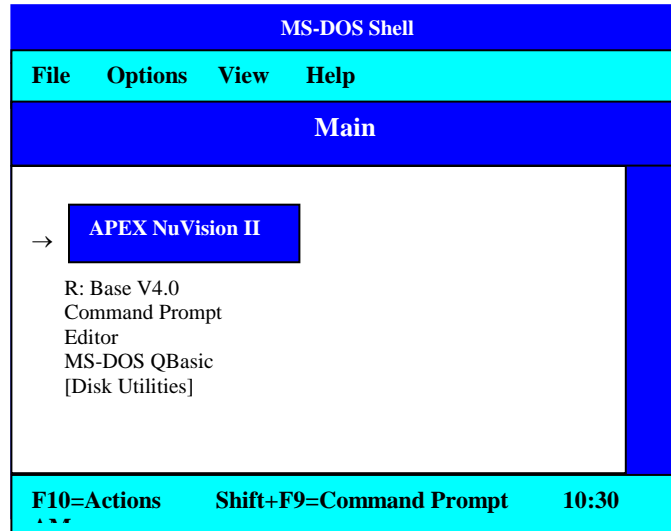
24. The following screen will appear on the monitor:

NuVision II Main Menu V03.05.001			
Applications Menu	Utilities	Setup	Exit
<div style="border: 1px solid black; padding: 10px; text-align: center;"> <b>Caution</b>            You've Chosen Program Exit            Are You Sure You Want to Exit            The Program            At This Time?  <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px 15px;">Yes</div> <div style="border: 1px solid black; padding: 5px 15px;">No</div> </div> </div>			
<b>LIVE</b>		<b>STATUS</b>	
Position	0.0000 In	10:04:05 AM 04-28-1990	
Load	0 lbs	Funct. Position Hold	
		Status System Normal	

**No** will be highlighted. Use the arrow key to highlight **Yes**. Press [ENTER].

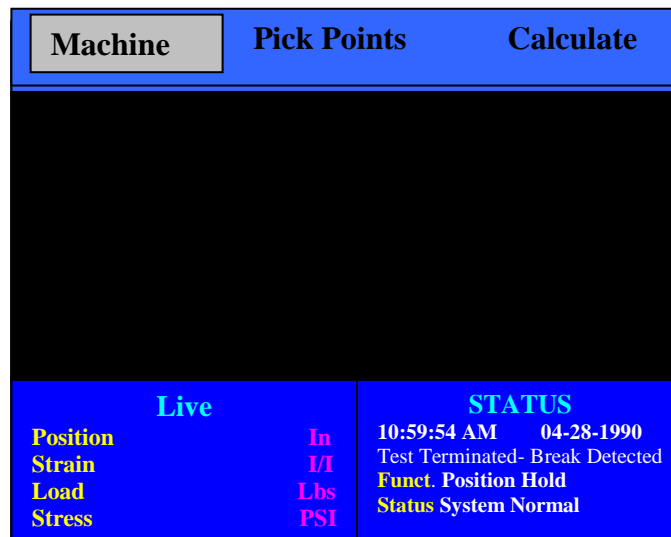
25. The following screen will appear on the monitor:





Press the [SHIFT] plus the [F9] key to return to the command prompt. The command prompt C:\> will appear and the computer, printer, monitor, and surge protector can be turned off.

26. **Note:** This step should be completed only if the test was terminated due to incorrect specimen insertion or insufficient preload. The following screen appears if the test is terminated:



**Machine** will be highlighted. Use the right arrow key to instead highlight **Calculate**. Press [ENTER]. The following screen will appear on the monitor:

<b>Machine</b>		<b>Pick Points</b>	<b>Calculate</b>
		<b>And Display Results&lt;ESC&gt;</b>	
<b>Live</b>		<b>STATUS</b>	
<b>Position</b>	<b>In</b>	10:59:54 AM 04-28-1990	
<b>Strain</b>	<b>I/I</b>	Test Terminated- Break Detected	
<b>Load</b>	<b>Lbs</b>	<b>Funct.</b> Position Hold	
<b>Stress</b>	<b>PSI</b>	<b>Status</b> System Normal	

**And Display Results<ESC>** will be highlighted. Then press [ENTER]. The following screen will appear on the monitor:

<b>Output</b>	Change Machine View Store Repeat
<b>WEARING SAFETY GLASSES</b> YES <b>Test</b> Gentest <b>Procedure</b> MAE 370 ALUMINUM <b>Test Date</b> 04-28-1990 <b>Test Time</b> 10:50:33 AM <b>Elapsed Time</b> 00:02:48 <b>Test Peak</b> 91550 PSI <b>Val@ Break</b> 7396 Lbs <b>Mod-Tangnt</b> 1225039 <b>Off-Yld-Std</b> PSI	
<b>Tested by</b> Ed Hopper <b>Test Counter</b> 00000200 <b>Datasets</b> 4219 <b>Area</b> 0.09375 In <sup>2</sup> <b>Val@ Break</b> 0.2339 I/I <b>EUL YIELD</b> PSI <b>Energy-Brk</b> 9596.9 In-Lbs	
<b>Live</b>	
<b>Position</b>	#
<b>In</b>	
<b>Strain</b>	#
<b>I/I</b>	
<b>STATUS</b>	
11:03:27 AM 04-28-1990	
Test Terminated- Break Detected	
Results & Datapoints Stored	
<b>Funct.</b> Position Hold	
<b>Status</b> System Normal	

**Output** will be highlighted. Use the right arrow key to instead highlight **Repeat**. Press [ENTER]. **Test Procedure <F9>** will be highlighted. Then press [ENTER]. The screen represented in step #10 will appear on the monitor. Return and repeat procedure starting with step #10.