Displacement Sensitivity in Remote-Objective-Speckle Metrology

by T.D. Dudderar, J.A. Gilbert, J.H. Bennewitz and M.A. Taher

ABSTRACT—In remote-objective-speckle (ROS) metrology an individual single-mode optical fiber (SMF) is used to illuminate a small surface region with coherent light. The coarse objective-speckle pattern which is reflected from the surface moves when the surface moves, and easily can be transmitted through a flexible multimode fiber bundle (MMB) back to a remote photoelectronic digitizer/computer system for analysis and correlation. Sensitivity is a function of both the intrinsic gain of the optical system operating between the MMB output and the photoelectronic digitizer and/or monitor, and the geometrical parameters of the illumination and observation system used to input the speckle into the MMB.

This investigation is concerned with understanding this latter, geometrical sensitivity, \( M \) for a fiber-based system. Results obtained for simple in-plane translations demonstrate the advantages and disadvantages of such an approach, and confirm the theoretical and experimental work of other investigators. For example, when the illumination beam is collimated, the speckle field moves with the surface such that \( M = 1 \). This is true regardless of the illumination angle, \( \theta \), or the observation distance, \( D \). However, when the illumination is not collimated (the numerical aperture of the SMF being greater than 0) the speckle field rotates as the surface translates and the geometrical sensitivity varies according to the two-parameter relationship

\[
M = \frac{\theta + Z}{\theta}
\]

where \( \theta \) is the radius of curvature of the illumination beam as it falls on the surface.

Introduction

Earlier studies by the authors\(^1\) demonstrate the application of fiber optics and photoelectronic-digitization/computer-correlation techniques to speckle metrology. Figure 1 shows a schematic diagram of the typical arrangement used in most of these studies. Here a single-mode fiber (SMF) is used to illuminate a small area on a remote test surface. Illumination of a small area of any diffusely reflecting surface\(^*\) gives rise to a three-dimensional field of random speckles associated with constructive and destructive interference of coherent light. The far-field speckle pattern has been classified as 'objective'\(^6\) (as opposed to 'subjective' speckles obtained by collecting the scattered radiation with a lens and focusing it on a recording medium). According to Weight\(^7\) these objective speckles are a random array of tapered cylinders whose transverse, \( L_x \), and axial, \( L_z \), dimensions are described by

\[
L_x = \lambda \frac{|Z|}{D}
\]

\[
L_z = \lambda \left( \frac{|Z|}{D} \right)^2
\]

Here \( Z \) is the observation distance from the surface and \( D \) is the width of the illuminated spot. A normal cross section of this field, as it falls on the input end of the multi-mode image bundle (MMB) positioned to sample the remote speckle field at \( Z \), provides a pattern of light and dark regions uniquely associated with the illuminated spot on the surface. Figure 2 shows such a speckle pattern as it could appear on the TV monitor. If the surface spot moves, the speckle pattern moves as well. Out-of-plane movements will be manifest in changes in the transverse size statistics of the speckles as described by \( L_x \), while all in-plane movements give rise to translations of the speckle pattern itself. Successive speckle patterns associated with various positions of a moving surface may thus be digitized, stored and subsequently analyzed by the computer to determine the actual movement (translation, rotation and/or deformation) of the illuminated surface. (Multiple surface locations may be measured by sequential illumination through an array of SMFs as desired.)

Of course, accurate interpretation of displacement depends on knowledge of the relationship between speckle movement as it appears (and is digitized) on the monitor screen, and the actual displacement of the remote test surface. In one interpretation, the various movements and properties of objective speckles are explained by assuming they were generated by a complex relationship between phase-dependent irregularities from different areas of the surface, such as might be produced by diffraction gratings.\(^*\) Objective-speckle motion was studied for simple displacements of an illuminated surface over a considerable range, and for the tilt about the center of the illuminated area around an axis parallel to the surface.\(^*\) It was noted that the type of surface is of major importance when the

\(^*\)A diffusely reflecting surface is a surface whose roughness dimension is large in comparison to the wavelength of light.
surface motion has a large tilt component. In many cases, individual bright speckles appeared and vanished as the angle of rotation was increased, thereby limiting the range of measurement. The first quantitative investigations of speckle movement caused by in-plane translation, in-plane rotation and tilt (rotation about an axis parallel to the object plane) were made in the diffraction plane (in a plane some distance away from the object). Later, shifts and small tilts around an axis parallel to the surface were studied by illuminating an object with collimated light and recording speckles in the Fourier plane of a lens system before and after tilt. A theory was developed and verified for the displacement and the structural changes in the diffraction field of an object translating uniformly.

Fig. 1—Schematic arrangement of the equipment used in studies of remote-objective-speckle metrology.

Fig. 2—Photograph of the objective-speckle pattern as it appears on the monitor screen.
under illumination by a gaussian beam. Finally, more general relationships were derived for objective-speckle motions in three-dimensional space resulting from displacements, rotations and deformations on a diffusely reflecting surface.  

In another interpretation it was assumed that objective speckles were formed by combinations of reflections scattered from mirror-like facets on the surface. Their movements were mathematically defined for rotations, transverse displacements, and tilts about an axis parallel to the scattering surface, illuminated using either collimated, diverging or converging illumination. Each speckle was considered to have a particular intensity and ‘shear phase’. Tests showed that no changes in the shear phase occurred for transverse surface movements and speckles slowly evolved due to the random variations in the intensity contributions from mirror facets. Tilt changed the shear phase to cause rapid decorrelation similar to that previously observed in Ref. 6.

A homologous-ray concept was also used to describe theoretically the speckle displacement in the diffraction field. Further studies show that defocused imaging gives rise to errors in the measurement of lateral translation in the case of non-collimated illumination and that speckle displacement in the defocused image is not only related to lateral translation but also to the rotation and strain of the object. Finally, changes in the structure of speckle in the diffraction field were mathematically described for an arbitrary combination of object translation, rotation and strain. These relations were partially verified by experiments using speckle photography and electronic speckle-correlation techniques, and later applied to develop a laser-speckle strain gage.

The present investigation is designed to show that objective-speckle movement can be measured through a flexible fiber-optic system using photoelectronic-digitization/computer-correlation techniques. Results obtained for simple in-plane translations demonstrate the feasibility of such an approach, and confirm the theoretical and experimental work of other investigators. In Fig. 1, the system magnification, $S$, (from the output end of the MMB through an imaging lens, $L_i$, into the vidicon camera tube and onto the monitor) depends on the system itself and may readily be calibrated for a given setup by laterally displacing either end of the MMB a known amount. (The MMB itself has an end-to-end magnification of unity, so it does not matter which end is moved.) However, the relationship between (a) the movement of the speckle as sampled by the input to the MMB at Z and (b) the transverse movement of the test surface itself is not unique. In fact, it depends on various parameters of the fiber-optic configuration used to generate and sample the objective-speckle field, as well as the movement of the surface itself.

In the simplest case, if the illumination is collimated (such as might be obtained by either using an unspeared beam directly from a laser, or by collimating the output from the SMF with a separate lens or lens system, the speckle field translates with the surface. Consequently, a unit displacement of the surface results in an identical unit displacement of the speckle field input to the MMB, regardless of where it is sampled by the MMB. It has been suggested that this ‘sensitivity’ can be altered by changing the incidence angle of the illumination beam. However, a simple experiment revealed that this is not the case for practical-normal or near-normal observation directions with collimated illumination. Figure 3 shows a plot of the measured shift in monitor raster columns vs. the applied lateral surface displacement in millimeters for collimated illumination at angles of 90 deg (normal to surface), 45 deg and 30 deg. (Below 30 deg there was not enough light reflected towards the MMB to provide a pattern of sufficient intensity to be useful.) For all three angles the data points are very nearly superimposed on one another. Therefore it may be concluded that with collimated illumination the objective-speckle field moves with the surface relatively independent of both the illumination direction, $\theta$, and the normal sample distance, $Z$.

However, it has been observed that whenever an unlensed SMF is used to provide an illumination beam, the overall system sensitivity appears to change as a function of both $Z$ and either the illuminated spot size or the SMF-to-surface distance, or both. Since the system configuration was essentially unchanged throughout most of these studies, these changes had to be the result of induced ‘rotations’ of the objective-speckle field. In many instances the speckle pattern sampled by the MMB translated much faster than did the point on the surface being illuminated, which represented a significant unaccounted for increase in sensitivity. Moreover, with the addition of lenses to provide beams of converging illumination it was observed that, depending on where the sampling was accomplished, the speckle pattern might be seen to (a) move rapidly in the opposite direction to the surface, (b) not move at all (just ‘boil’ while the surface moves), or (c) move very slowly in the same direction as the surface motion. Earlier efforts to interpret this behavior in terms of the effective numerical aperture, $N_A$, (or half angle of divergence) of the illumination beam, the diameter, $D$, of the illuminated spot on the test surface and the MMB sampling distance, $Z$, were relatively unsuccessful owing primarily to insufficient accuracy in determining $D$ and $N_A$ using unlensed SMF illumination.  

In this study, the objective speckle motion has been examined using unlensed SMFs of two independently determined $N_A$s, and considering only the two parameters $Z$ and the wavefront radius of curvature, $\rho$, rather than $Z$, $N_A$, and $D$.

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*If the sampling plane (face of the MMB) is inclined at an angle $\alpha$ from the normal to the test surface it will resolve the cos component of the objective-speckle-field motion at that point, be it translating, rotating, or whatever.*

*As suggested by K.-A. Suteen.*
Experiment

Two series of tests were run using the configuration shown in Fig. 1 with the test surface mounted on a micro-meter-driven translation stage. In the first series of tests an SMF of NA = 0.105 was positioned at five different illuminating distances from the test surface (1.20 mm, 1.60 mm, 2.40 mm, 15.90 mm and 25.40 mm as measured to an accuracy of ±0.5 mm). For a fixed sampling distance of Z = 101.6 mm, an overall sensitivity, S, in raster columns of speckle-shift mm of lateral surface displacement was determined for each illumination distance by carefully translating the test surface laterally far enough to move the speckle field on the monitor 100-digitized raster lines, noting the required surface translation, ΔX, and converting. Since the core of the SMF is only around 6-7 μm in diameter, the illumination distance was taken to be a reasonable estimate of the radius of curvature, ρ, of the illuminating wave front as it strikes the surface. Table 1 gives the overall sensitivity, S, vs. q for these tests, along with the results for a single test run with the SMF replaced by an unspread beam direct from the laser such that q = ∞ (collimated illumination). In these tests the system magnification, S, was field calibrated to give 58.6 raster columns of movement on the monitor for each millimeter of lateral translation at the MMB. Subsequently, a second series of tests were run using an SMF of NA = 0.119 at eight different distances (1.5 mm, 3.1 mm, 6.1 mm, 12.2 mm, 24.4 mm, 48.8 mm, 76.2 mm and 101.6 mm, again as measured to an accuracy of ±0.5 mm) and four more sampling distances. As listed in Table 2, these ranged from 50.8 mm up to 812.8 mm, the greatest sampling distance for which there was sufficient intensity to generate worthwhile speckle patterns in all q's. Since the system had been rebuilt and refocused it was recalibrated to yield a system magnification, S, of 53.93 col/mm.

Finally, a series of observations were made at q = 101.6 mm in which an iris diaphragm was used to independently aperture the illumination beam. During these runs no changes in sensitivity were seen over a ten-fold change in the spot size, D. This indicates that D alone has no influence on the sensitivity, although D and the aperture (NA or whatever) together define the radius of curvature, ρ, which, it will be shown, does influence the sensitivity.

Analysis and Discussion

In each case the total sensitivity, S, defining the raster columns of speckle shift per millimeter of lateral surface displacement, was converted to objective-speckle magnification, M, in millimeters of speckle movement per millimeter of lateral surface displacement by dividing by the appropriate system sensitivity, S. The resulting values of M are given in Tables 1 and 2 and plotted against q in Fig. 4. In these data the objective speckle magnification ranges from near unity at q = ∞ to almost 600 at q = 1.5 mm and Z = 812.8 mm.

The data shown in Fig. 4 as a log-log plot form an array of almost linear, equally spaced curves which fan out slightly as q → 0 and M increases. Moreover, as q → ∞, where it would be expected that M → 1 and the curves would flatten, some decrease in slope is evident in the data taken closer to the surface (at smaller Zs).

Figure 5(a) provides a simplified geometrical description of the speckle-field movement appropriate to model this data. As the surface moves laterally a distance ΔX, the illumination reflected to the sample plane at Z moves a

**TABLE 1**

<table>
<thead>
<tr>
<th>q (mm)</th>
<th>S (col/mm)</th>
<th>M* (mm/mm)</th>
<th>M (q+Z) (mm/mm)</th>
<th>ω (mm⁻¹)</th>
<th>q+Z (mm)</th>
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<td>1.2 ± 0.50</td>
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<td>67.2</td>
<td>0.652</td>
<td>86.4</td>
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<tr>
<td>25.4 ± 0.50</td>
<td>305</td>
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<tr>
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<td>61</td>
<td>1.04</td>
<td>0.000</td>
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</tr>
</tbody>
</table>

*M = S/Ss, where Ss = 58.6 columns/mm
ω = M(q+Z)

**TABLE 2**

<table>
<thead>
<tr>
<th>q (mm)</th>
<th>S (col/mm)</th>
<th>M* (mm/mm)</th>
<th>M (q+Z) (mm/mm)</th>
<th>ω (mm⁻¹)</th>
<th>q+Z (mm)</th>
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<td>12.2 ± 0.50</td>
<td>296</td>
<td>5.3</td>
<td>0.064</td>
<td>5.2</td>
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<tr>
<td>24.4 ± 0.50</td>
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<td>3.1</td>
<td>0.041</td>
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<td>0.020</td>
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</tr>
<tr>
<td>76.2 ± 0.50</td>
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<td>—</td>
<td>1.7</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>101.6 ± 0.50</td>
<td>ND</td>
<td>—</td>
<td>1.5</td>
<td>183</td>
<td></td>
</tr>
</tbody>
</table>

*M = S/Ss, where Ss = 53.93 columns/mm
ω = M(q+Z)
ND: not determined because the objective-speckle size was too small to be resolved by the system (less than 0.01 mm)
distance \( \Delta X' \). It is as if the speckle field rotates about \( P' \), a virtual image of the point source of illumination at \( P \). The location of \( P' \) is shown in the figure by the dotted lines converging behind the test surface. Defining the objective-speckle magnification, \( M \), as \( \Delta X'/\Delta X \), by geometry we may compute

\[
M = \frac{(q + Z)}{q}
\]  

(2)

A similar relationship was derived by Gregory and Yamaguchi. Alternatively, a field parameter, \( \omega \), may be defined by dividing \( M \) by the total reflected distance from the point source image behind the surface to \( Z \), giving

\[
\omega = M/(q + Z)
\]

(3)

In other words, \( \omega \) should equal the curvature, \( 1/q \). Tables 1 and 2 list values of \( \omega \) computed for all the data taken in this study. This analysis represents a reasonable first-order description of the geometry of objective-speckle-field movement with noncollimated illumination, regardless of the actual statistics of the interference contributing to the speckle formation itself. As evidence of this, Fig. 6 shows a log-log plot of \( \omega \) vs. \( q \). Here it can clearly be seen that at larger \( q \) the data groups closely along a single straight line of slope \( \omega = -1 \), although there is some spread in values as \( q \to 0 \). However, considering the uncertainty in determining \( q \), together with the limited resolution (0.0025 mm) of the micrometers used to translate the test surface, this scatter at small \( q \) (large \( M \)) is quite moderate and probably does not represent any significant departure from the trend established at larger \( q \) (smaller \( M \)). Figure 7 shows a log-log plot of \( (q + Z)/q \) vs. \( M \) which exhibits the same excellent correlation at the lower values with some spread as \( M \) becomes large. Consequently, for all data \( \omega \) may reasonably be taken to be inversely proportional to \( q \); the simple relation given by eq (2) is valid for at least a first-order estimate of \( M \).

Furthermore, eq (2) also indicates that if \( q \) is negative, denoting a converging rather than diverging illumination beam, the sign of \( M \) will change as \( Z \) varies from less to more than \( |q| \). Consider Fig. 5(b), comparable to 5(a) but drawn for a converging rather than diverging illumination. In this case it is not clear whether \( M \) varies directly or otherwise with \( \omega \). Consequently, different regions of the three-dimensional objective-speckle field which forms in front of the test surface rotate with or against the surface motion as it pivots about \( P' \). At \( Z \) less than \( |q| \) the magnification, \( M \), is positive and the speckle movement will be in the same direction as the surface movement but reduced, \( M < 1 \). At \( Z = |q| \), \( M \) becomes zero and the speckle pattern will be seen to change without translating, while as \( Z \) grows larger than \( |q| \), \( M \) becomes negative and increases rapidly—explaining the retrograde speckle movements observed earlier. In addition, if a divergent beam is used to illuminate the test surface at some angle, \( \beta \), the center of objective-speckle rotation, \( P' \), will be symmetrically positioned at \( -\beta \) behind the surface. To the first order, the rotation will resolve the sin \( \beta \) component of the surface movement. On the other hand, if the illumination beam is convergent, the point-
source image point, \( P^* \), will lie in front of the surface at \(-\beta\). Again, the rotation will resolve the \( \sin \beta \) component of the surface movement, but from the new origin at \( P^* \). In either case the apparent sensitivities will be altered as suggested.\(^\text{22}\)

Finally, in the earlier study described in Ref. 4 an attempt was made to relate the total observed sensitivity, \( S \), in cols/mm (raster columns of speckle shift per millimeter of surface motion) to the three parameters \( D \) (spot size), \( NA \) (numerical aperture) and \( Z \). These data may be correlated more effectively by using the two-parameter system for estimating the speckle magnification \( M \) represented by eq (2). In order to accomplish this the overall sensitivities, \( S \), given in Ref. 4 were converted to \( M \) values by dividing by the appropriate system sensitivity, \( S_0 = 58.6 \) col/mm. Then estimates of the radii of curvature were computed from \( D \) and \( NA \) using the relation

\[
\hat{\phi} = \frac{D}{2NA}
\]

Equation (2) was then used to compute an 'estimated' objective-speckle magnification to be compared with the \( M \) values determined above. These results, along with the original data as published in Ref. 4 with typographical errors corrected, are given in Table 3. Figure 8 shows a log-log plot of the magnitudes of \((\hat{\phi} + Z)/\hat{\phi}\) vs. \( M \) which, again considering the intrinsic uncertainties in the process for determining \( \hat{\phi} \) and \( S \), shows a reasonably robust correlation over several orders of magnitude, and for both diverging and converging illumination including positive and retrograde speckle movements!

**Conclusions**

The following conclusions should be made for objective-speckle sensitivity.

1. The use of a collimated illumination beam provides a measuring scheme for in-plane translation that is essentially independent of the distances between the source of illumination, the test surface and the sampling plane. However, this arrangement provides for no optical gain (\( M = 1 \)) and is, consequently, less sensitive.

2. The use of a noncollimated illumination beam provides for a measuring scheme whose sensitivity depends on both the radius of curvature of the beam at the surface and the distance to the objective-speckle-field sampling plane. This results from an optical-gain relationship given by eq (2) describing a rotation of the objective-speckle field about an 'image' of the point source of illumination. This point of rotation lies at \( P^* \) behind the test surface for divergent illumination (\( \phi \) positive), and at \( P^* \) in front of the test surface for convergent illumination (\( \phi \) negative).

3. It may appear that this rotating objective-speckle field model described by eq (2) suggests that either

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**Fig. 7**—Log-log plot of \((\hat{\phi} + Z)/\hat{\phi}\) vs. the objective-speckle magnification, \( M \)

**Fig. 8**—Log-log plot of \((\hat{\phi} + Z)/\hat{\phi}\) vs. \(|M|\) as calculated from the three-parameter data of Bennewitz et al.\(^\text{6}\). Here the solid symbols represent positive values and the open symbols represent negative (retrograde) values.

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### Table 3

<table>
<thead>
<tr>
<th>$\mathcal{Q}$</th>
<th>$D/2NA$</th>
<th>$N_A$</th>
<th>$D$</th>
<th>$S$</th>
<th>$M^*$</th>
<th>$\frac{\mathcal{Q} + Z}{\mathcal{Q}}$</th>
<th>$S$</th>
<th>$M$</th>
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<th>$M$</th>
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<td>3.175</td>
<td>38</td>
<td>0.8</td>
<td>0.3</td>
<td>-264.0</td>
<td>-4.5</td>
<td>-4.4</td>
<td>-781.0</td>
<td>-13.3</td>
<td>-12.4</td>
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<tr>
<td>$-18.9$</td>
<td>0.042</td>
<td>1.588</td>
<td>107</td>
<td>1.8</td>
<td>-116.0</td>
<td>-1.6</td>
<td>-1.6</td>
<td>-472.0</td>
<td>-9.8</td>
<td>-9.1</td>
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<tr>
<td>$-9.5$</td>
<td>0.794</td>
<td>433</td>
<td>7.4</td>
<td>6.4</td>
<td>-1791.0</td>
<td>-22.5</td>
<td>-22.5</td>
<td>-5630.0</td>
<td>-98.1</td>
<td>-54.7</td>
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</tbody>
</table>

*M* = $S/\omega_s$, where $\omega_s = 58.6$ col/mm.

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Acknowledgments

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