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SPECKLE METROLOGY COMBINED WITH FINITE-ELEMENT MODELING FOR STRESS ANALYSIS

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Abstract. The work described in this report represents a successful demonstration of a hybrid approach to the analysis of structural deformation. Mathematical modeling was incorporated into the process of reducing data from a digital correlation analysis of experimentally obtained speckle data. These experimental data were collected by both directly imaging the speckled surface of the test subject onto a vidicon camera/digitizer system and transmitting this image to the camera via a flexible coherent fiber optic image bundle. Considerable savings of time and resources can be realized through applications of this hybrid approach in which the strengths of the theoretical and experimental procedures complement each other beautifully. The final hybrid results compare very favorably with values obtained by both a theoretical mathematical (finite-element) analysis and an independent experimental (high frequency moire) study, demonstrating the accuracy and reliability of this hybrid procedure. Finally, the successful use of flexible optical fiber elements for data access demonstrates the potential for application of speckle hybrid techniques to the study of remote or otherwise inaccessible regions of a prototype structure.

Subject terms: speckle; speckle metrology; hybrid stress analysis; finite-element analysis; digital image correlation; fiber optic applications.

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CONTENTS

1. INTRODUCTION

This paper describes the successful demonstration of a hybrid stress analysis technique using speckle metrology designed to achieve a more efficient, more accurate solution by combining experimental and analytical techniques. A critical region of a prototype structure is modeled using finite elements, and experimental data taken from the correlation of digitized speckle patterns obtained from the boundaries of the same region on the prototype during loading are used to specify values at the "nodes" of the finite-element model. Potentially, this approach can enhance efficiency by (1) significantly reducing computational requirements and (2) providing a much closer match to the actual boundary conditions as defined by loading the prototype structure. Moreover, such hybrid techniques may yield solutions where neither experimental nor analytical methods alone will succeed.

The present work parallels a comparable successful effort to demonstrate a different hybrid technique using the moire method as described in an earlier report.[1] However, the present speckle hybrid technique has major advantages over the earlier moire hybrid technique in that (1) it does not require the preparation and attachment of a precision ultrahigh pitch grating to the surface of the prototype test subject, (2) it facilitates remote access via fiber optic imaging, (3) its correlation analysis is computer-based, and (4) it may readily be automated to feed data directly into the finite-element routine for rapid analysis.

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2. THE HYBRID APPROACH

The idea of a hybrid approach to structural analysis is not new.[2–4] Although there is a long history of development for engineering studies based on mathematical models (finite-difference equations, boundary value integrals, finite-element methods) and on experimentally determined data (gaging, optical or acoustical metrology, etc.), each method has well-known limitations. Mathematical studies depend on the correspondence between some abstract model subjected to specified boundary conditions and a real structure subjected to complex interacting forces. A detailed model quickly becomes very large and mathematically complex, placing large burdens on the computational and financial resources of the designer. Also, the validity of the results depends on how well boundary conditions have been incorporated into the model. On the other hand, direct experimental methods of analysis often yield only a few data values, measured at isolated points on the structure. Acquiring these data in certain critical regions may be difficult or dangerous (e.g., complex structures, inaccessible locations, hazardous environments). In such cases, fiber optics might be used to provide safe, effective access for appropriate data acquisition.

The hybrid method takes advantage of the strengths of the mathematical/theoretical and experimental approaches, while minimizing their weaknesses. Basically, the idea is to drastically reduce the required finite-element mesh (to improve efficiency and reduce computation time) and to incorporate measured values instead of generalized boundary conditions (to improve accuracy), while simultaneously gaining the capacity to study relatively inaccessible regions of a structure through the use of fiber optic components.

3. SPECKLE METROLOGY

Surface motion studies using speckle photography (as opposed to speckle interferometry[5]) usually fall into one of two basic categories: (1) optical methods that generate displacement-related fringe patterns[6] and (2) photoelectronic-digital methods that use numerical correlation techniques.[7] The former approach has the advantage of simplicity, but unavoidable primary* and secondary† speckle size considerations (e.g., to be detectable the surface displacement must be greater than the characteristic speckle size) limit its range and resolution. Furthermore, double-exposure photographic techniques must be employed, limiting measurement to displacements between two successive states. On the other hand, digitized speckle techniques use photoelectronic recording, digitization, and computer-based numerical correlation, which obviate the need for photographic work and optical processing. In addition, these methods have a greater inherent range of measurement because many speckle size considerations are relaxed; e.g., correlations may be obtained for the measurement of displacements both larger and *smaller* than the characteristic speckle size. Even more important may be the capacity of digital correlation techniques to operate effectively over a wide range of speckle sizes ranging from a significant fraction of the field of view down to the resolution limit of the system. In remote applications, these limits may be defined by the size and resolution characteristics of the fiber optic imaging bundle.[7] In the present study these various considerations prompted the use of a whole field artificial or "white light" speckle technique[8] whose data could be transmitted reliably by a relatively low resolution image bundle and correlated without a significant loss of either resolution or range.

Finally, with photoelectronic recording a sequence of many speckle fields may be digitized and correlated in succession to provide a displacement history (or velocity field) if desired, even for highly unsteady motion.

4. THE TESTS

To compare the speckle hybrid method with a purely numerical approach, tests were carried out on a simple linear elastic structure using both methods. A notched beam to be loaded in symmetric three-point bending was both machined of PSM-1 photoelastic plastic (see Fig. 1) and modeled with a finite-element mesh [see Fig. 2(a)]. By virtue of the symmetry of the problem, it was necessary to analyze only half the beam. Appropriate boundary conditions, together with the mesh, were input to the finite-element program (ANSYS) running on a mainframe computer. The ANSYS program produced a listing of the displacements at all the nodal points and a plot of the maximum shear stress intensity values throughout the entire half-model. This plot is shown in Fig. 3, where the stress intensity contours represent multiples of 3.45 MPa (500 psi). Assuming that trust is placed in the mesh used to model the notched beam and that the specified boundary

*Primary speckle refers to the speckle associated with the surface whose motion is to be measured.

†Secondary speckle refers to the speckle noise that appears in the diffraction halo as a result of the coherent illumination used to interrogate the specklegram.

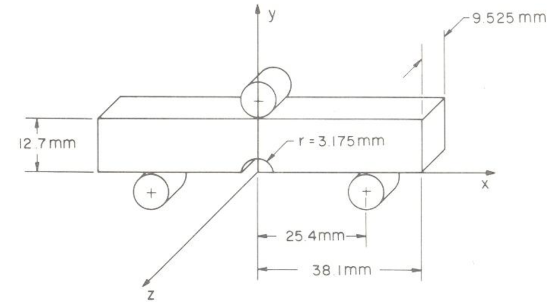

Fig. 1. Notched beam used for comparing standard finite-element method with hybrid approach to analysis.

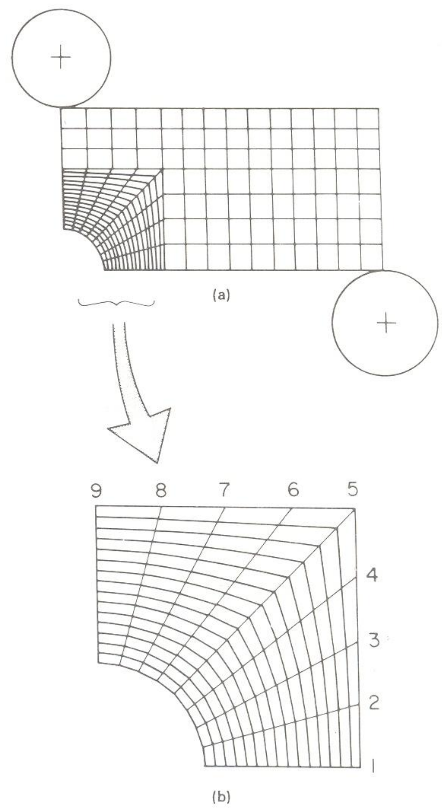

Fig. 2. (a) Finite-element mesh used to model the notched beam of Fig. 1. Because of symmetry, only half the beam has to be modeled. (b) Subregion of mesh around the notch in the beam. Numbers indicate locations (nodes) chosen for displacement analysis using speckle correlation technique.

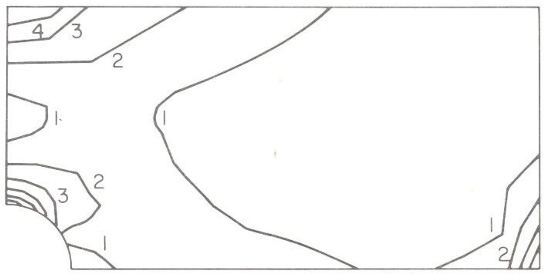

Fig. 3. Stress intensity plot for the notched beam under load. Values are calculated by standard finite-element method. Stress intensity contours represent multiples of 3.45 MPa (500 psi).

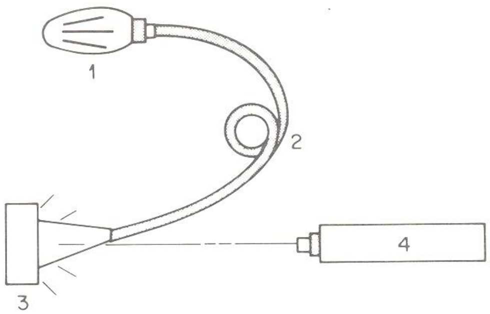

Fig. 4. Experimental setup for direct imaging of speckled surface of beams onto vidicon camera/digitizer: (1) tungsten lamp, (2) incoherent fiber optic bundle (illuminator), (3) test surface with "artificial" speckle, (4) vidicon camera.

conditions accurately describe the loadings, the problem may be considered solved, yielding theoretical elastic displacements and stresses throughout the beam.

For the experimental part of the hybrid test, the surface of the PSM-1 prototype beam was spray-painted white and spattered with black paint to produce a suitable random pattern of artificial speckle. It was then mounted in a loading frame where it could be loaded and deformed as desired. To simulate an idealized three-point bending, three metal rollers were used to provide transverse "line" loadings, one on top of the beam directly above the notch and the other two on the bottom, one positioned 25.4 mm on each side of the notch. Loading was accomplished by raising the lower rollers 254 μm to bend the beam upward around the top roller, which immobilized the center of the top of the beam above the notch. The speckled surface was then illuminated with the light from a tungsten filament lamp transmitted through a small flexible optical fiber bundle (see Fig. 4). The reflected light was imaged into a vidicon camera (Hamamatsu C1000), and the collected image was digitized into a 256 × 256 pixel array, with intensity levels distributed over an 8-bit gray scale.

A subset of the finite-element mesh that enclosed the critical notched area of the beam was chosen for numerical analysis. Nine locations around this subregion, corresponding to the nodal points on the boundary of this minimized finite-element mesh or subset, were selected [see Fig. 2(b)], and displacements at these locations were determined experimentally. The full-field speckle patterns obtained from this region before and after deformation of the PSM-1 beam were digitized and submitted to a correlation routine. The routine used in this study performed a two-dimensional correlation and assumed that the displacements were homogeneous over small areas. No corrections were made for possible rotation or warping. If these types of displacement are anticipated, more complex routines, such as those developed by Peters et al.,[9] may be used. However, this routine did incorporate Lagrangian weighting functions to allow interpolation of subpixel displacements. The horizontal and vertical displacements u and v, respectively, measured at the nine selected locations are given in columns 2 and 3 of Table I.

TABLE I. Horizontal and vertical displacement values u and v for selected locations (nodes) on the beam from speckle data obtained by direct imaging of the test surface onto the vidicon (see Fig. 4). These were calculated by speckle correlation and inserted into the finite-element program to yield the hybrid results shown in Fig. 5(b). Displacement values u′ and v′ were obtained by remote imaging through a fiber optic bundle (see Fig. 6) and yielded the hybrid results shown in Fig. 5(c).

Node	u (mm)	v (mm)	u′ (mm)	v′ (mm)
1	0.055	0.058	0.050	0.053
2	0.042	0.057	0.037	0.057
3	0.023	0.054	0.028	0.051
4	0.008	0.057	0.014	0.058
5	0.001	0.055	−0.001	0.057
6	0.000	0.037	0.000	0.040
7	0.000	0.021	0.002	0.027
8	0.000	0.011	0.002	0.018
9	−0.001	0.008	−0.001	0.014

These nodal displacement values and the subregion mesh were submitted to the ANSYS program for analysis. For comparison, an enlarged view of the maximum shear stress intensity contours determined by a standard finite-element approach and given earlier as the lower left corner of Fig. 3 is shown in Fig. 5(a) for the subregion of interest. The ANSYS output plot of maximum shear stress intensity values determined by the hybrid approach using the experimentally measured nodal displacements for the same region is shown in Fig. 5(b). As before, the contours represent multiples of 3.45 MPa (500 psi). The close agreement between the two plots for the region around the notch is obvious, and the largest maximum shear stress intensity at the root of the notch was calculated by ANSYS as 25.0 MPa (3617 psi) using the standard finite-element analysis and as 25.4 MPa (3679 psi) using the hybrid approach, a difference of only 2%.

A third test was conducted in which a coherent optical fiber bundle was used to transmit images of the illuminated speckled test surface to the vidicon camera (see Fig. 6). The entire speckle analysis procedure was repeated, and the resulting displacement values u′ and v′ measured at the selected nodal points are given in columns 4 and 5 of Table I. Minor differences between the u and v displacements given in columns 2 and 3 of Table I and those given in columns 4 and 5 are not unexpected, since the model was fully unloaded and reloaded for the latter test. These new values were used as input to the ANSYS program, and the resulting shear stress plot is shown in Fig. 5(c). The same comments may be made for these results as were made for those obtained with the "direct image" analysis; minor irregularities are evident around the

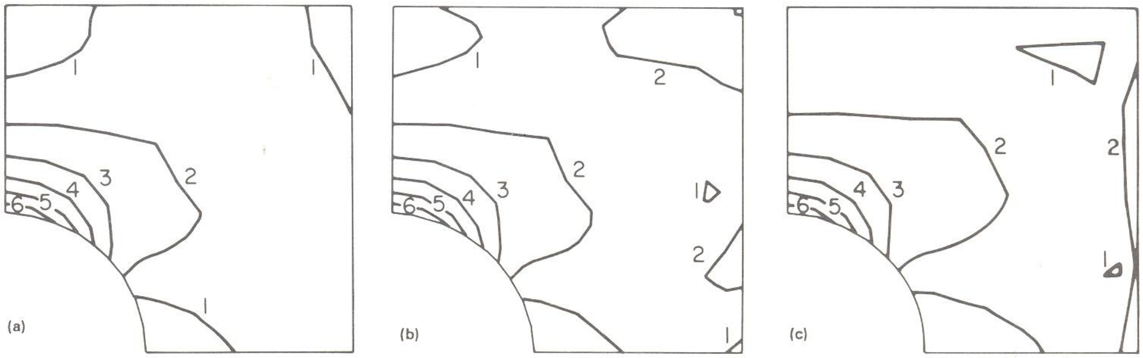

Fig. 5. Stress intensity plots:(a) for subregion around notch. This is an enlargement of part of Fig. 3. Contours represent multiples of 3.45 MPa (500 psi); (b) for subregion around notch as calculated by ANSYS using nine measured values of nodal displacement. Contours are labeled as in (a); (c) for subregion around notch from data obtained through coherent fiber optic bundle. Contours are labeled as in (a).

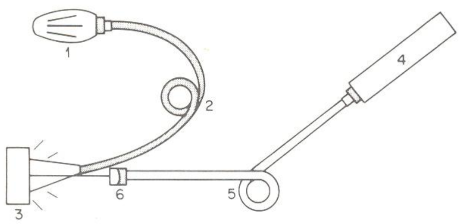

Fig. 6. Experimental setup with speckle images transmitted to vidicon camera/digitizer through coherent fiber optic image bundle: (1) tungsten lamp, (2) incoherent fiber optic bundle (illuminator), (3) test surface with "artificial" speckle, (4) vidicon camera, (5) coherent fiber optic bundle, (6) imaging lens.

boundary (due to the variations in the nodal input values), but in the critical neighborhood of the notch, agreement with the two previously obtained plots [Figs. 5(a) and 5(b)] is excellent. The hybrid approach gave a maximum stress intensity value of 24.7 MPa (3573 psi) at the notch root, which again is within 2% of the value obtained from the standard finite-element analysis.

Finally, it should be noted that had the experimental loadings been less "ideal," e.g., had the rollers been made of plastic instead of metal and the beam been made of metal instead of plastic, then as the load was increased, the rollers would have deformed plastically, the "line" loadings would have been lost, and the finite-element analysis would have become increasingly inaccurate. On the other hand, neither hybrid technique would be in any way adversely affected by this development, and each could be expected to provide just as accurate an evaluation of the conditions at the root of the notch as under those (perhaps more realistic) loading conditions as was demonstrated here for the more "idealized" loading conditions.

5. SUMMARY

The excellent agreement between the results obtained from a standard finite-element analysis and the results obtained from a hybrid approach demonstrates the validity of the latter, while the much simpler mesh and the incorporation of actual data values demonstrate its potential to simplify computations and increase confidence in the results.

Although a full finite-element analysis has advantages for the initial design of a structure and allows the specification of overall system parameters, it is cumbersome and expensive when the task is to analyze local anomalies in an existing structure. In this latter case, the hybrid approach can be a much simpler and more cost-effective way to solve problems or to study the response of prototypes.

In this project, the field data on displacement were obtained using artificial speckle techniques.[8] These data were acquired both by direct imaging and by image transmission through a flexible optical fiber bundle, demonstrating capacity for remote access. The excellent results obtained here and in an earlier study[1] using a moire-based hybrid analysis demonstrate that the power of the hybrid approach is not in the particular measurement technique utilized, but in the incorporation of experimental data values obtained from a test of the actual structure (or a model of the structure) into a finite-element routine.

Finally, it should be noted that in these tests it was not possible to make a wholly experimental evaluation of the strain field at the root of the notch because, as in many real situations, the displacements in the critical region were too small to measure with the accuracy needed to yield reliable displacement gradients. Nevertheless, experimental displacement measurements made at enough points around the boundary of the region of interest gave values sufficient to accurately determine the strain distribution around the notch root by a hybrid numerical (finite-element) procedure. On the other hand, for many real problems in stress analysis where similar conditions preclude purely experimental evaluation, complicated loadings, geometry, or local material conditions may also render purely numerical modeling methods ineffective or excessively difficult. If, however, suitable experimental data can be acquired, many such nearly untractable problems might be readily analyzed by hybrid techniques.

6. ACKNOWLEDGMENTS

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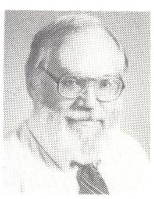

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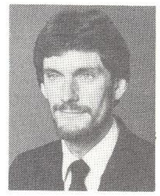

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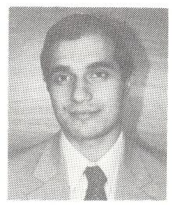

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