TWO-DIMENSIONAL STRESS ANALYSIS COMBINING HIGH-FREQUENCY MOIRÉ MEASUREMENTS WITH FINITE-ELEMENT MODELING


In the hybrid approach presented here, experimental results obtained from high-frequency moiré patterns provide data to be input at nodes on the boundary of a finite-element mesh in the neighborhood of a critical subsection; e.g., surrounding a stress concentration, and stresses are evaluated throughout this region with the finite-element routine. The hybrid method completely analyzes the experimental results while, at the same time, reduces the size of the required finite-element mesh and associated computational time and expense. Boundary conditions are not required for the finite-element routine because their effects are contained in the experimentally measured displacements.

A simple structure was used to demonstrate the technique. For comparison purposes the stress distribution was initially determined throughout the entire structure by finite-element analysis and, to lend additional credibility to the finite-element results, the maximum stress was hand calculated based on strength of materials. Displacements were then evaluated on an experimental prototype at nodal locations surrounding the highly stressed, critical region of the structure. Hybrid-stress analysis was performed in this area with a mesh that represented only a small portion of that used in the initial finite-element analysis. Results of this test agreed to within a few percent of the values obtained from finite-element analysis and the hand calculation, clearly demonstrating the hybrid method to be an effective alternative to finite-element modeling, with the advantages that, (1) the method can be applied to predict the real behavior of a structure in cases where, unlike the present example, the applied loads are too complex to define from a mathematical standpoint and that, (2) computational requirements are significantly reduced. While the present demonstration was not carried out on a personal computer, certainly the latter advantage would greatly facilitate such an approach, especially if realized using the ANSYS PC/LINEAR routines now available.

FINITE-ELEMENT MODELING

The commercially available finite-element routine used for this demonstration is called ANSYS (Engineering Analysis System). It can be programmed to generate a very complex mesh constructed from a variety of different types of elements. When appropriate material properties, applied loads and boundary conditions are specified, ANSYS analyzes the mesh and solves for nodal displacement, stress, and reaction forces, providing output in a variety of forms.

The symmetrical notched beam shown in Fig. 1 was used as the demonstration structure. Equal displacements were prescribed at each of the lower supports so that only one-half of the part had to be modeled (loading and geometry were symmetrical). This significantly reduced computational time and associated costs. Quadrilateral elements were selected for the mesh. (They are ideal for the analysis of two-dimensional problems involving solid bodies of constant thickness.) Two orthogonal degrees of freedom, which ultimately correspond to displacement outputs called UX and UY in the program, existed at each node.

Figure 2a shows that, except in the notched region, relatively coarse elements were used throughout most of the two-dimensional finite-element model. Around the notches a more sophisticated mesh was generated with a progressively finer mesh approaching the highly stressed area. The dimensions were specified as shown in Fig. 1, and the material properties were taken to be those of FSM-1 photoelastic plastic which has an elastic modulus of 2344 MPa (340 ksi) and a Poisson's ratio equal to 0.38. An upward displacement of 7.62 microns was prescribed at the lower support and ANSYS was used to generate the shear-stress contour plot shown in Fig. 2b. Figure 2a shows an enlargement of the critical region bounded by the lines labeled AB and BC on Fig. 2a, while Fig. 3a shows the maximum shear-stress contours labeled in multiples of 29.3 kPa (4.25 psi).

Verification of the maximum stress at the root of the notch was accomplished by hand calculation using the flexure...
Fig. 1—Notched beam in three-point bending

Fig. 2a—Finite-element mesh for the notched beam

Fig. 2b—Maximum shear-stress contours for the notched beam generated from the finite-element analysis

Fig. 3a—Enlarged view of the notched region

Fig. 3b—Maximum shear-stress contours in the critical region of the notched beam generated from the finite-element analysis alone

Fig. 3c—Maximum shear-stress contours in the critical region of the notched beam generated from the hybrid analysis
formula from strength of materials' equations. The effect of the notch was taken into account by determining the stress-concentration factor using standard curves developed from empirically obtained data. The calculations yielded a maximum shear stress of 327.5 kPa (47.5 psi).

HIGH-FREQUENCY MOIRÉ

High-frequency moiré (also called moiré interferometry or interferometric moiré) is a highly sensitive full-field optical method introduced by Post and used to measure two-dimensional surface displacement. The technique requires the deposition of a high-frequency phase-type diffraction grating on the surface of a specimen. The interference of this specimen grating with a virtual reference grating having twice the spatial frequency (created in space by interfering two plane wavefronts) yields a moiré pattern governed by:

$$U = (1/f)n$$

where $U$ is the displacement component measured perpendicular to the lines in the reference (also called virtual or master) grating, $f$ is the spatial frequency (the reciprocal of the distance between lines in the reference grating, usually defined as its pitch, $p$), and $n$ is the fringe order number observed in the moiré pattern. Any two systems of non-parallel gratings (usually oriented perpendicular to one another) can be used to completely determine the displacement vector in the plane tangent to each point on a surface.

THE HYBRID METHOD

The hybrid stress-analysis method uses experimentally determined displacements (measured from high-frequency moiré patterns taken from the part in question, as described above) as boundary conditions for a reduced numerical stress-analysis routine. The associated finite-element mesh represents only a small portion of that previously used for complete numerical analysis in which boundary conditions were specified at nodes located along the external boundary of the specimen. In the proposed hybrid example, experimentally measured displacements were prescribed along the boundary of the subsection at locations that are really internal nodes (1-24) of the original finite-element mesh. The nodes located along the free boundary of the specimen were specified to be traction free.

EXPERIMENTAL EVALUATION

A high-frequency moiré analysis was performed on the surface of the prototype in order to evaluate displacements at nodes surrounding the critical subsection shown in Fig. 3a. First a custom mold was made by doubly exposing a high resolution photographic plate (Kodak 125-02) to an in-space virtual grating of 1200 lines/mm (created by interfering two coherent, plane wavefronts of argon ion laser light at an angle equal to 36 degrees). The photographic plate was rotated 90 deg between exposures to record two orthogonal gratings and was processed using common darkroom procedures. It was then bleached in aqueous bromine and dipped in a wetting agent (Kodak photoflow) prior to air drying. The gelatin of the plate shrinks upon drying, but shrinkage is restrained locally in the zones occupied by the silver compound in the emulsion, resulting in nonuniform shrinkage and a regularly corrugated mold on which to form a crossed-line specimen grating.

An aluminum film was vacuum deposited onto the mold to yield a highly reflective cross grating approximately 100 angstroms thick, which was then transferred to the surface of the notched beam with an epoxy adhesive (PC-10C, Measurements Group). The Kodak photoflow used in processing the photographic plate which forms the mold plays an important role in this process as a release agent since it leaves, on the dry-emulsion surface, a very thin uniform film of contaminant which interferes with the adhesion between the mold and the aluminum.

Figure 4 shows the experimental set-up used to record high-frequency moiré patterns. Here, a laser beam (wavelength equal to 514.5 nm) is guided by a monomode fiber to the focal point of a parabolic mirror (0.4-mm diameter, 2.0-m focal length) to produce a collimated beam. Half the beam strikes the specimen directly (beam A), while the other half is reflected by a plane mirror (beam B). These two beams produce a reference grating of frequency $f = 2400$ lines/mm at the surface of the sample: this grating interacts with the specimen grating of frequency $f/2$. Light from the +1 and -1 diffraction orders of the specimen grating is intercepted by an imaging lens, while light from all other orders is diffracted away. The moiré interferometry pattern carried by these two beams can be observed and photographed in the image plane of the lens. Additional details of the experimental technique can be found in the report by Post.

In practice, the grating on the surface of the specimen is never composed of perfectly straight, uniformly-spaced corrugations. Even if interrogated by a perfect reference grating, a few fringes, due to aberrations, misalignment, and/or preload, would be observed in the field. In addition, since the wavefronts of the two interfering beams of the reference grating are never perfectly plane either, the reference grating also contributes to this initial pattern. These initial (no-load) fringes can be subtracted out from the full-load fringe orders by introducing carrier patterns into the field and observing the resulting moiré fringes. Fringe contrast can be enhanced by optical filtering.

Figures 5a and 5b show moiré patterns of $U$ and $V$ displacements, respectively (measured along the $X$ and $Y$ axes shown in Fig. 1), obtained when both lower supports were displaced upward 7.82 microns. In practice, these fields were produced in two separate operations. Using the crossed-line specimen grating, one set of lines was aligned with the reference grating, and fringes
of one displacement field were photographed. Then the specimen was unloaded and rotated through 90 deg. Loads were applied a second time and the fringes of the orthogonal displacement field were photographed. Figures 6a and 6b show the result of optically filtering these patterns in order to increase fringe contrast. Absolute fringe order numbers were established and displacements were analyzed using eq. (1).

Table 1 lists the displacements used for hybrid analysis along with the node numbers and locations at which they were prescribed (see Fig. 3a). The number of carrier fringes was insufficient to produce a moiré in the V-displacement pattern in the highly strained regions near the notch and below the center loading point (see Figs. 5b and 6b), even though these displacements were well above the resolution of the measurement technique. Therefore, this component of displacement could not be specified at node 8 and the 16 nodes (9-24) along the vertical center line. This demonstrates that the response at the notch could not have been determined by experimental measurements alone. Fortunately, this missing data has little effect on the solution obtained when using the hybrid method, since the nodes in question can be left to displace freely in the vertical direction. The equations in the finite-element routine ensure that continuity is maintained in that area of the mesh and a two-dimensional stress distribution can be evaluated throughout the critical region. Figure 3c shows the resulting maximum shear-stress contours labeled in multiples of 29.3 kPa (4.25 psi).

**RESULTS**

The maximum shear-stress contours plotted in Figs. 3b and 3c (determined by using the conventional finite-element analysis and the hybrid technique, respectively) agree to within a few percent of all points in the critical region. The wholly numerical approach predicts a maximum shear stress of 372.5 kPa (54 psi) at the notch (node 24) while the hybrid method shows 355 kPa (51.5 psi) at the very same location. Considering that some experimental data is missing and that the U and V displacement patterns were recorded in two separate
TABLE 1—NODAL DISPLACEMENTS

<table>
<thead>
<tr>
<th>Node Number</th>
<th>X Location (mm)</th>
<th>Y Location (mm)</th>
<th>U Displacement (mm)</th>
<th>V Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.94</td>
<td>0.00</td>
<td>1.51 × 10⁻³</td>
<td>3.20 × 10⁻³</td>
</tr>
<tr>
<td>2</td>
<td>7.94</td>
<td>1.98</td>
<td>1.13 × 10⁻³</td>
<td>3.12 × 10⁻³</td>
</tr>
<tr>
<td>3</td>
<td>7.94</td>
<td>3.97</td>
<td>7.45 × 10⁻⁴</td>
<td>3.06 × 10⁻³</td>
</tr>
<tr>
<td>4</td>
<td>7.94</td>
<td>5.95</td>
<td>4.03 × 10⁻⁴</td>
<td>3.02 × 10⁻³</td>
</tr>
<tr>
<td>5</td>
<td>7.94</td>
<td>7.94</td>
<td>4.03 × 10⁻⁵</td>
<td>3.02 × 10⁻³</td>
</tr>
<tr>
<td>6</td>
<td>5.95</td>
<td>7.94</td>
<td>3.22 × 10⁻⁴</td>
<td>2.62 × 10⁻³</td>
</tr>
<tr>
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<td>7.94</td>
<td>2.63 × 10⁻⁵</td>
<td>2.86 × 10⁻³</td>
</tr>
<tr>
<td>8</td>
<td>1.98</td>
<td>7.94</td>
<td>3.22 × 10⁻⁸</td>
<td>Not specified</td>
</tr>
<tr>
<td>9 to 24</td>
<td>0.00</td>
<td>7.94 to 3.17</td>
<td>0.00</td>
<td>Not specified</td>
</tr>
</tbody>
</table>

*Covering fifteen 0.318-mm Y (vertical) decrements

tests, these results are excellent. In addition, the value of maximum shear stress predicted by the hybrid analysis comes closer to that obtained by hand calculation (327.5 kPa or 47.5 psi) than the value obtained from pure finite-element modeling. These results demonstrate that the hybrid approach is an effective alternative to whole-body finite-element modeling, having the advantages that computational requirements are significantly reduced, and that the method can be applied to predict the real behavior of an essentially two-dimensional problem in cases where the applied loads and/or the structure itself are otherwise too complex to allow a simple numerical description.

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REFERENCES