Diffraction properties of substrate guided-wave holograms

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Abstract. Substrate guided-wave (SGW) holograms, recorded and reconstructed using light waves guided through optical substrates, are currently being incorporated into visual displays, optical integrated circuits, and nondestructive testing systems. In this paper, a coupled wave approach is used to analyze the diffraction properties of SGW holograms. The expressions obtained allow SGW holograms to be compared with conventional transmission and reflection holograms. The analysis is restricted to cases in which holograms are recorded and reconstructed using plane waves. The object beam is also assumed to be at normal incidence to the hologram. Results are presented for holograms recorded on a silver halide emulsion and illuminated using a krypton laser. The study shows that SGW holograms have higher diffraction efficiencies under the Bragg condition, higher angular selectivities, and more moderate wavelength selectivities than holograms recorded using conventional means.

Subject terms: substrate guided-wave (SGW) holograms; Bragg condition; coupled wave theory; volume holograms.

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1 Introduction

Substrate guided-wave (SGW) holograms are recorded by guiding the reference wave to the hologram using an optical substrate. As compared to conventional holographic recording systems, SGW systems are compact, portable, robust, and easy to align. Since the illuminating beam is encapsulated within the substrate and only the holographic image is diffracted, the hologram can be viewed close to the recording plane without the danger of eye damage associated with the undiffracted light produced during off-axis reconstruction. These inherent advantages have led to a number of practical applications for SGW holograms ranging from illuminators in spatial light modulators to sensors in holographic nondestructive testing systems.

In general, holograms are classified as either transmission or reflection; the classification depends upon the recording geometry. Figure 1(a), for example, illustrates that a transmission hologram is recorded when the object and reference waves impinge on the same side of the recording medium, whereas a reflection hologram, Fig. 1(b), is recorded when the beams impinge from opposite sides. In conventional holography, the recording beams propagate in free space before entering the recording material, and the incident angle of the reference wave, $\theta_{\text{air}}$, can range between 0 and 180 deg with respect to the positive z axis. Refraction occurs at the air-emulsion boundary, and for angles different from 0 and 180 deg, the incident angle within the emulsion, $\theta_\epsilon$, is reduced. For holograms recorded on silver halide, the range 0$\leq\theta_\epsilon\leq37.84$ deg corresponds to a transmission hologram, whereas the range 142.2$\leq\theta_\epsilon\leq180$ deg corresponds to a reflection hologram; the angular range 37.8$\leq\theta_\epsilon\leq142.2$ deg is inaccessible when using these conventional holographic recording geometries.

The inaccessible range can be accessed using the SGW holographic recording geometries illustrated in Fig. 2. The range 37.8$\leq\theta_\epsilon\leq90$ deg in Fig. 2(a) corresponds to an SGW transmission hologram, whereas the range 90$\leq\theta_\epsilon\leq142.2$ deg in Fig. 2(b) corresponds to an SGW reflection hologram. Since they are recorded in an angular range that is different from that ordinarily associated with holographic recording,
Fig. 1 The object beam is at normal incidence in most practical holographic recording systems. Recording geometries for (a) transmission and (b) reflection holograms.

relatively little attention has been paid to defining the diffraction properties of SGW holograms.

This paper addresses this issue by studying the diffraction properties of the holograms generated with the recording geometries shown in both Figs. 1 and 2. The analysis is restricted to cases in which holograms are recorded and reconstructed using plane waves. The object beam is also assumed to be at normal incidence to the hologram, a characteristic common to the majority of holograms designed for practical applications ranging from display to interferometry. Other recording geometries, more often used to produce diffraction gratings, holographic mirrors, or filters, make it awkward to view the reconstructed image and, in the case of holographic nondestructive testing, complicate displacement analysis.

Since the object beam remains in the same position relative to the hologram, diffraction characteristics can be plotted as a function of the reconstruction angle. By choosing realistic values for the recording parameters, direct comparisons can be made between SGW holograms and their conventionally recorded counterparts.

2 Prior Related Research

Conventionally recorded and SGW holograms are special cases of volume holograms. The coupled wave theory, presented by H. Kogelnik, is the most common approach applied to understand the diffraction characteristics of such a hologram. In this approach, expressions that represent the wavefronts propagating within the hologram are substituted into the inhomogeneous wave equation to obtain a pair of coupled wave equations. These equations are solved analytically by making several approximations and by applying appropriate boundary conditions. Results have been presented for conventional transmission and reflection holograms, albeit in a relatively abstract fashion; however, little attention has been paid to cases in which the recording waves impinge on the hologram at relatively oblique angles (i.e., for SGW holograms).

In the ensuing discussion, the process of obtaining the diffracted wave from a volume hologram via the coupled wave theory is reviewed. The diffraction characteristics of SGW holograms are examined using the results, and these characteristics are compared with those obtained for conventional transmission and reflection holograms.

3 The Coupled Wave Theory

Figure 3(a) shows a volume hologram generated by superimposing an object and a reference beam. In formulating the
Figure 3(b) shows the processed hologram reconstructed using a plane wave polarized in the TE mode and incident at angle $\theta_i$, where, in general, $\theta_i \neq \theta_r$. The light wave propagating inside the hologram, $\Psi(x,z)$, can be characterized by the inhomogeneous Helmholtz equation as

$$\nabla^2 \Psi + k_2^2 \left[ 1 + \frac{\varepsilon_2 \cos(K \cdot r)}{\varepsilon_1} \right] \Psi = 0, \text{ where } k_2 = \frac{2\pi}{\lambda_2} \sqrt{\varepsilon_1}. \tag{3}$$

In Eq. (3), $k_2$ is the average propagation constant inside the hologram, and $\lambda_2$ is the wavelength of the illuminating beam; in general, $\lambda_2 \neq \lambda_1$.

The solution to Eq. (3) is assumed to be the sum of two waves: a transmitted wave corresponding to the undiffracted light, and a diffracted wave corresponding to the minus one diffraction order. Mathematically, this assumption is expressed as

$$\Psi(x,z) = \Psi_{e0}(z) \exp[j(k_0 x + k_0 z)] + \Psi_{e-1}(z) \exp[j(k_{-1} x + k_{-1} z)], \tag{4}$$

where $\Psi_{e0}(z)$ and $\Psi_{e-1}(z)$ are the complex envelopes of the two plane waves that propagate along vectors $k_0$ and $k_{-1}$, respectively. The scalar components of the propagation vectors in Eq. (4) are given by

$$k_{0x} = k_2 \sin \theta_i, \quad k_{0z} = k_2 \cos \theta_i,$$

$$k_{-1x} = k_2 \sin \theta_i - K \sin \phi, \quad k_{-1z} = k_2 \cos \theta_i - K \cos \phi. \tag{5}$$

Equations (4) and (5) are referred to as the coupled wave assumption. By substituting Eq. (4) into Eq. (3) and using the orthogonality of the exponential functions, the following pair of linear differential equations is obtained:

$$\frac{\partial^2 \Psi_{e0}}{\partial z^2} + 2j k_0 \frac{\partial \Psi_{e0}}{\partial z} + k_2^2 \Psi_{e0} = 0, \tag{6}$$

$$\frac{\partial^2 \Psi_{e-1}}{\partial z^2} + 2j k_{-1} \frac{\partial \Psi_{e-1}}{\partial z} + k_2^2 \Psi_{e-1} = 0, \tag{7}$$

where

$$\Theta = \frac{k_0^2 - k_{-1}^2}{2k_2} \tag{8}$$

is called the dephasing factor and $\Theta$ is zero when the Bragg condition is met. That is, when

$$\theta_i = \theta_r \quad \text{and} \quad k_2 = k_1 \quad \text{(or} \quad \lambda_2 = \lambda_1). \tag{8}$$

The dephasing factor is a measure of the deviation in angle and wavelength between the recording and reconstructing waves, and when slight deviations occur (for example, $\theta_i = \theta_r + \Delta \theta$ or $k_2 = k_1 + \Delta k$), it can be expressed as

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\[ \Theta = k_1 [\sin(\theta_r - \theta_0)] \Delta \theta + 2 \left( \sin^2 \frac{\theta_r - \theta_0}{2} \right) \Delta k. \] (9)

The second-order linear partial differential equations in Eq. (6) are the coupled wave equations that represent the interrelation between the transmitted and diffracted waves inside the hologram. The properties of the diffracted wave can be analyzed by solving these equations for \( \Psi_{-1}(z) \).

To solve the coupled equations analytically, it is usually assumed that the complex envelopes \( \Psi_{\epsilon}(z) \) and \( \Psi_{-\epsilon}(z) \) are slowly varying functions of \( z \), such that
\[
\frac{\partial^2 \Psi_{\epsilon}}{\partial z^2} \ll \left| \frac{\partial \Psi_{\epsilon}}{\partial z} \right| \quad \text{and} \quad \frac{\partial^2 \Psi_{-\epsilon}}{\partial z^2} \ll \left| \frac{\partial \Psi_{-\epsilon}}{\partial z} \right| .
\] (10)

Under this assumption, the second-order derivatives can be neglected in Eq. (6), thereby reducing the governing equations to a first-order form. They can be rewritten as
\[
\frac{\partial \Psi_{\epsilon}}{\partial z} = \frac{j \kappa}{C_{\epsilon \epsilon}} \Psi_{\epsilon-1},
\]
\[
\frac{\partial \Psi_{\epsilon-1}}{\partial z} = \frac{j \Theta}{C_{-\epsilon \epsilon}} \Psi_{\epsilon-1} + \frac{j \kappa}{C_{-\epsilon \epsilon}} \Psi_{\epsilon 0},
\]
where
\[
C_{\epsilon \epsilon} = \frac{k_0}{k_2}, \quad C_{-\epsilon \epsilon} = \frac{k_0}{k_2}, \quad \kappa = \frac{k_0 k_2^2}{4e_\epsilon}.
\] (12)

In Eqs. (11) and (12), the quantities \( C_{\epsilon \epsilon} \) and \( C_{-\epsilon \epsilon} \), are referred to as slant factors. If the Bragg condition given by Eq. (8) is satisfied,
\[ C_{\epsilon \epsilon} = \cos \theta_r \quad \text{and} \quad C_{-\epsilon \epsilon} = \cos \theta_0 . \] (13)

Since the discussions are limited to cases in which the illuminating wave deviates only slightly from the Bragg condition, the relations in Eq. (13) are assumed to hold true.

Boundary conditions, corresponding to the hologram recording geometry, must be specified to solve Eq. (11). In general, holograms can be classified as either transmission or reflection; SGW holograms also fall into these two categories. The type of hologram under consideration can be identified by introducing the parameter
\[ C = \frac{C_{\epsilon \epsilon}}{C_{-\epsilon \epsilon}} = \frac{\cos \theta_r}{\cos \theta_0} . \] (14)

Transmission holograms have \( C > 0 \), whereas reflection holograms have \( C < 0 \).

Figure 4(a), for example, shows the reconstruction of a transmission hologram recorded with the object wave at normal incidence. The illuminating wave used to reconstruct the hologram is incident on the plane \( z = 0 \) with a complex envelope \( \Psi_{\epsilon 0}(d) \). Since the diffracted wave passes through the opposite side of the plate at \( z = d \), it has zero amplitude on the plane \( z = 0 \). Hence, the boundary conditions can be expressed as
\[ \Psi_{\epsilon 0}(0) = \Psi_{\epsilon 0}(0) \quad \text{and} \quad \Psi_{\epsilon -1}(0) = 0 . \] (15)

Figure 4(b), on the other hand, illustrates that when a reflection hologram is illuminated, the diffracted wave emerges through the same side on which the hologram is illuminated. The illuminating wave is incident on the plane \( z = d \) with a complex envelope \( \Psi_{\epsilon 0}(d) \), whereas the diffracted wave has zero amplitude on the plane \( z = 0 \). The corresponding boundary conditions are
\[ \Psi_{\epsilon 0}(d) = \Psi_{\epsilon 0}(d) \quad \text{and} \quad \Psi_{\epsilon -1}(0) = 0 . \] (16)

The diffracted waves from transmission and reflection holograms are obtained by solving Eq. (11) using the boundary conditions given by Eqs. (15) and (16), respectively. The solutions are

transmission:
\[
\Psi_{\epsilon -1}(d) = j \Psi_{\epsilon 0}(0) \left( C_{\epsilon \epsilon} \right)^{1/2} \sin \left( \frac{\xi^2 + \nu^2}{2} \right) \left( 1 + \frac{\xi^2 \nu^2}{2} \right)^1 e^{j \xi } ,
\]

reflection:
\[
\Psi_{\epsilon -1}(d) = \Psi_{\epsilon 0}(d) \left( C_{\epsilon \epsilon} \right)^{1/2} \frac{1}{j(\xi/\nu) + (1 - \xi^2/\nu^2)^{1/2} \coth(\nu^2 - \xi^2)^{1/2}} .
\] (17)
where

\[
\xi = \frac{\Theta d}{2C_{-1}z} \quad \text{and} \quad \nu = \frac{k d}{(\pm C_{0e}C_{-1}z)^{1/2}}.
\]

In Eq. (17), the + and − in the ± sign hold for transmission and reflection holograms, respectively.

Equations (9), (12), and (13) can be substituted into Eq. (17) to obtain expressions for the diffracted waves when the illuminating conditions deviate slightly from the Bragg condition. The results are

transmission:

\[
\Psi_{e-1}(d) = j \Psi_{e1}(d) \left( \frac{\cos \theta}{\cos \theta_0} \right)^{1/2} \sin \left( \frac{\xi}{2} + \frac{\nu^2}{2} \right) e^{j \xi}.
\]

reflection:

\[
\Psi_{e-1}(d) = \Psi_{e1}(d) \left( \frac{\cos \theta}{\cos \theta_0} \right)^{1/2} \times \frac{1}{j \left( \xi / \nu + (1 - \xi^2 / \nu^2) \right) \coth \left( \nu^2 - \xi^2 \right)^{1/2}}.
\]

When the Bragg condition is satisfied (\( \xi = 0 \)), these expressions simplify to

transmission:

\[
\Psi_{e-1}(d) = j \Psi_{e1}(d) \left( \frac{\cos \theta}{\cos \theta_0} \right)^{1/2} \sin \left( \frac{k d \Delta \theta}{\cos \theta_0} + 2 \frac{d \Delta k}{\cos \theta_0} \sin \frac{\theta - \theta_0}{2} \right),
\]


reflection:

\[
\Psi_{e-1}(d) = \Psi_{e1}(d) \left( \frac{\cos \theta}{\cos \theta_0} \right)^{1/2} \tanh \left( \frac{k d \Delta \theta}{\cos \theta_0} \right),
\]

Equations (18) and (19) form the foundation for the ensuing discussions regarding the diffraction properties of SGW holograms. Before these discussions begin, however, it is important to examine their validity, especially with respect to the slowly varying approximation.

4 The Validity of the Theory

In solving the coupled wave equations, Eq. (10) was applied and second-order derivatives were neglected. When the Bragg condition is satisfied, the diffracted waves emerging from the output plane, \( z = d \), are given by Eq. (19). Substituting these expressions into Eq. (10) gives rise to the following inequalities:

transmission:

\[
v \tan \nu < k d \cos \theta_0,
\]

reflection:

\[
v \tanh \nu < k d \cos \theta_0,
\]

where \( v \) is defined in Eq. (18).

The left- and right-hand sides of the expressions in Eq. (20) are plotted against \( \theta_r \) in Fig. 5. Data for these plots were compiled for \( \theta_0 = 0 \), assuming that holograms were recorded and illuminated using a krypton laser with a wavelength \( \lambda_1 = 647 \text{ nm} \) on AGFA 8E75 silver halide plates having a 7-\( \mu \text{m} \)-thick emulsion and an index of refraction of \( n = \varepsilon_1 \varepsilon_2 / 2 \). The degree of modulation in permittivity was assumed to be \( \varepsilon_2 = 0.05 \). The angular ranges \( 0 \leq \theta_r \leq 37.84 \text{ deg} \) and \( 142.2 \leq \theta_r \leq 180 \text{ deg} \) correspond to conventional transmission and reflection holograms, respectively, whereas the angular ranges \( 37.8 \leq \theta_r \leq 90 \text{ deg} \) and \( 90 \leq \theta_r \leq 142.2 \text{ deg} \) correspond to SGW transmission and reflection holograms, respectively.

Assuming that the inequalities in Eq. (20) hold when the value of the left-hand side is 10 times less than that of the right-hand side, the upper limit of \( \theta_r \) for a transmission hologram is 82.5 deg, whereas the lower limit of \( \theta_r \) for a reflection hologram is 90.3 deg.

In the following arguments, the diffraction efficiency is calculated assuming that the Bragg condition is satisfied; therefore, results are only considered reliable in the ranges \( 0 \leq \theta_0 \leq 82.5 \text{ deg} \) and \( 90.3 \leq \theta_0 \leq 180 \text{ deg} \). It is also assumed that results over these ranges are valid in cases where only slight deviations from the Bragg condition occur; however, the extent of these deviations may significantly affect the validity.

\[
\text{Fig. 5 Validity considerations for the coupled wave theory with the following recording parameters: } \lambda_1 = 647 \text{ nm}, \ d = 7 \mu \text{m}, \ n = \varepsilon_1 \varepsilon_2 / 2 = 1.63, \ \text{and } \varepsilon_2 = 0.05.
\]
5  Diffractive Characteristics of Volume Holograms

From a physical standpoint, a volume hologram can be treated as a set of semireflective interfaces that form due to the periodic variation of permittivity. When the hologram is illuminated, a portion of the incident light is diffracted by these interfaces. The undiffracted light passes through the interfaces, remaining parallel to the direction of illumination. The diffracted wave is characterized by its diffraction efficiency, defined as

$$\eta = \frac{|C_{-12}|}{C_{0c}} \frac{\Psi_{-1} \Psi_{0}^{*} - \Psi_{-1}^2}{\Psi_{ei}^2}.$$  (21)

Equation (21) indicates that the diffraction efficiency is the ratio of the output (diffracted) to input (illuminating) energy flows measured normal to the input and output planes of the hologram. Since the normal component is computed by multiplying the incident energy flow by the cosine of the angle of incidence, the brightness of an image reconstructed from a hologram depends on the reconstruction angle and the diffraction efficiency. In general, holograms having higher diffraction efficiencies are more desirable.

Maximum diffraction efficiency can only be achieved when the hologram illumination satisfies the Bragg condition; the efficiency changes when the illuminating beam deviates from this condition. The degrees to which changes in efficiency occur due to deviations in angle and wavelength are referred to as the angular and wavelength selectivity, respectively. Thus, a complete understanding of the diffraction in a volume hologram requires a comprehensive study of diffraction efficiency, angular selectivity, and wavelength selectivity.

5.1  Diffraction at Bragg Incidence

As mentioned previously, holograms for display and interferometry are usually recorded with the object wave at normal incidence (see Figs. 1 and 2). When the information recorded in these holograms is reconstructed by a plane wave at Bragg incidence, the diffracted waves are characterized by Eq. (19). By substituting these results into Eq. (21), we obtain

transmission:  \[ \eta = \frac{k_{i} \varepsilon_{d} d}{4 \varepsilon_{i} (\cos \theta_{0})^{2}} \]  (22)

reflection:  \[ \eta = \tanh^{2} \left[ \frac{k_{i} \varepsilon_{d} d}{4 \varepsilon_{i} (\cos \theta_{0})^{2}} \right] \]

Figure 6, plotted using the recording parameters \( \lambda_{1} = 647 \text{ nm}, d = 7 \mu \text{m}, n = \varepsilon_{i}^{1/2} = 1.63 \) and \( \varepsilon_{d} = 0.05 \), illustrates that the diffraction efficiency \( \eta \) varies as a function of the incident angle of the reference wave, \( \theta_{0} \). The solid line represents the rigorous mathematical solution, and the dotted line is an approximation in the area in which the theory is known to be invalid. To the extent that the theory is considered valid (0.5 0.6 5 180 deg), the curve shows an increase in efficiency as \( \theta_{0} \) approaches 90 deg, suggesting that an SGW hologram is more efficient than a conventional hologram.

Figure 6  Diffraction efficiency for a volume hologram versus the incident angle of the reference beam. Recording parameters are listed in Fig. 5; the solid line represents the rigorous mathematical solution, while the dotted line is an approximation in the area in which the theory is known to be invalid.

5.2  Angular Selectivity

To study the angular selectivity, it is assumed that the hologram is illuminated with a plane wave having the same wavelength (i.e., \( \Delta k = 0 \)), but at a slightly different angle from that of the reference wave used to record the hologram. When the expressions for the diffracted waves, given in Eq. (18), are substituted into Eq. (21), we obtain

transmission:  \[ \eta = \frac{\sin^{2}(\xi^{2} + \nu^{2})}{1 + \xi^{2} / \nu^{2}} \]

reflection:  \[ \eta = \frac{1}{1 + (1 - \xi^{2} / \nu^{2}) / \sinh^{2}(\nu^{2} - \xi^{2}) / \nu^{2}} \]

where

\[ \xi = \frac{k_{i} d}{2} \Delta \theta \sin \theta_{0} \]

\[ \nu = \frac{k_{i} \varepsilon_{d}}{4 \varepsilon_{i}} d (\pm \cos \theta_{0}) \]

Figures 7(a) and 7(b) show plots of the diffraction efficiency \( \eta \) versus the angular deviation \( \Delta \theta \) for transmission and reflection holograms, respectively. Initially \( \eta \) decreases as \( \Delta \theta \) increases; the angular range over which the efficiency changes from a maximum to its first minimum is defined as the angular width. The angular width \( \Delta \theta_{w} \) is shown in Fig. 7(a) for the curve corresponding to \( \theta_{0} = 75 \text{ deg} \). The angular width is an indication of the angular selectivity of the hologram; the smaller the angular width, the higher the angular selectivity.

Expressions for the angular width may be obtained by setting the arguments of the sine and hyperbolic-sine func-
Figure 8 shows the angular width plotted as a function of the reference beam angle. The solid line represents the rigorous mathematical solution, while the dotted line is an approximation in the area in which the theory is known to be invalid. To the extent that the theory is considered valid, the plot shows that SGW holograms are more angular-selective than conventional holograms. This observation could be important, for example, when designing an angular division multiplex hologram for optical information storage. In this case, a hologram having a smaller angular width would allow more information to be stored.

5.3 Wavelength Selectivity

To study the wavelength selectivity, it is assumed that the hologram is illuminated with a plane wave having no angular deviation (i.e., \( \Delta \theta = 0 \)), but with a wavelength slightly different from that of the reference wave used to generate the hologram. When the expressions for the diffracted waves, given in Eq. (18), are substituted into Eq. (21) with \( \theta_0 = 0 \) and \( k_1 = \frac{2 \pi (\varepsilon_1)^{1/2}}{\lambda_1} \), we obtain

transmission: \( \eta = \frac{\sin^2(\xi^2 + \nu^2)^{1/2}}{1 + \xi^2/\nu^2} \),

reflection: \( \eta = \frac{1}{1 + (1 - \xi^2/\nu^2)/\sinh^2(\nu^2 - \xi^2)^{1/2}} \),

where

\[ \xi = - \frac{d k_1 \Delta \lambda}{\lambda_1} \frac{\sin^2 \theta_r}{2} \] and \( \nu = \frac{(k_1 \varepsilon_2/4 \varepsilon_1) d}{\sqrt{\pm \cos \theta_r}} \).
Figures 9(a) and 9(b) show plots of the diffraction efficiency $\eta$ versus the wavelength deviation $\Delta \lambda$ for transmission and reflection holograms, respectively. Initially $\eta$ decreases as $\Delta \lambda$ increases; the angular range over which the efficiency changes from a maximum to its first minimum is defined as the wavelength width. The wavelength width $\Delta \lambda_w$ is shown in Fig. 9(a) for the curve corresponding to $\theta_r = 75$ deg. The wavelength width is an indication of the wavelength selectivity of the hologram; the smaller the wavelength width, the higher the wavelength selectivity.

Expressions for the wavelength width may be obtained by setting the arguments of the sine and hyperbolic-sine functions in Eq. (25) equal to $\pi$ and then solving for $\Delta \lambda_w$ as follows:

transmission:

$$\Delta \lambda_w = \frac{\lambda_1}{\sin^2(\theta_r/2)} \left[ \left( \frac{\pi}{k_1 d} \right)^2 - \left( \frac{\varepsilon_2}{4\varepsilon_1 (\cos\theta_r)^{1/2}} \right)^2 \right]^{1/2},$$

reflection:

$$\Delta \lambda_w = \frac{\lambda_1}{\sin^2(\theta_r/2)} \left[ \left( \frac{\pi}{k_1 d} \right)^2 + \left( \frac{\varepsilon_2}{4\varepsilon_1 (-\cos\theta_r)^{1/2}} \right)^2 \right]^{1/2}. \quad (26)$$

Figure 10 shows the wavelength width plotted as a function of the reference beam angle. The solid line represents the rigorous mathematical solution, while the dotted line is an approximation in the area in which the theory is known to be invalid. To the extent that the theory is considered valid, the plot shows that an SGW transmission hologram is more wavelength-selective than a conventional transmission hologram; however, an SGW reflection hologram is less wavelength-selective than a conventional reflection hologram. These observations could be important, for example, when fabricating a display hologram designed to be illuminated with white light. In this case, a hologram having a smaller wavelength width would produce less chromatic aberration.

6 Conclusion

The diffraction properties of volume holograms have been analyzed in an effort to understand the unique diffraction characteristics associated with SGW holograms. To this end, a coupled wave theory was applied to predict the diffracted
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waves produced by volume holograms illuminated by plane waves. In performing the calculations, it was assumed that the holograms were recorded with the object beam at normal incidence; this condition is most useful for three-dimensional image display and simplifies displacement analysis in industrial measurement applications. The fact that the object beam remained in the same position relative to the hologram made it possible to plot the validity conditions, diffraction efficiency, angular selectivity, and wavelength selectivity as a function of the angle used to reconstruct the hologram.

Numerical results, presented for $\lambda_1 = 647$ nm, $d = 7$ $\mu$m, $n = \varepsilon_1^{1/2} = 1.63$, and $\varepsilon_2 = 0.05$, show that the theory is invalid when the incident angle of the reference beam is in the vicinity of 90 deg. Maximum diffraction efficiency is achieved only when the reconstructing beam satisfies the Bragg condition; the diffraction efficiency initially decreases as the angular and/or wavelength deviation increases.

By observing the changes in the diffraction efficiency as a function of angle, it is concluded that SGW holograms are more efficient and angular-selective than their conventional counterparts. SGW transmission holograms are more wavelength-selective than conventional transmission holograms, whereas SGW reflection holograms are less wavelength-selective than conventional reflection holograms.

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References


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