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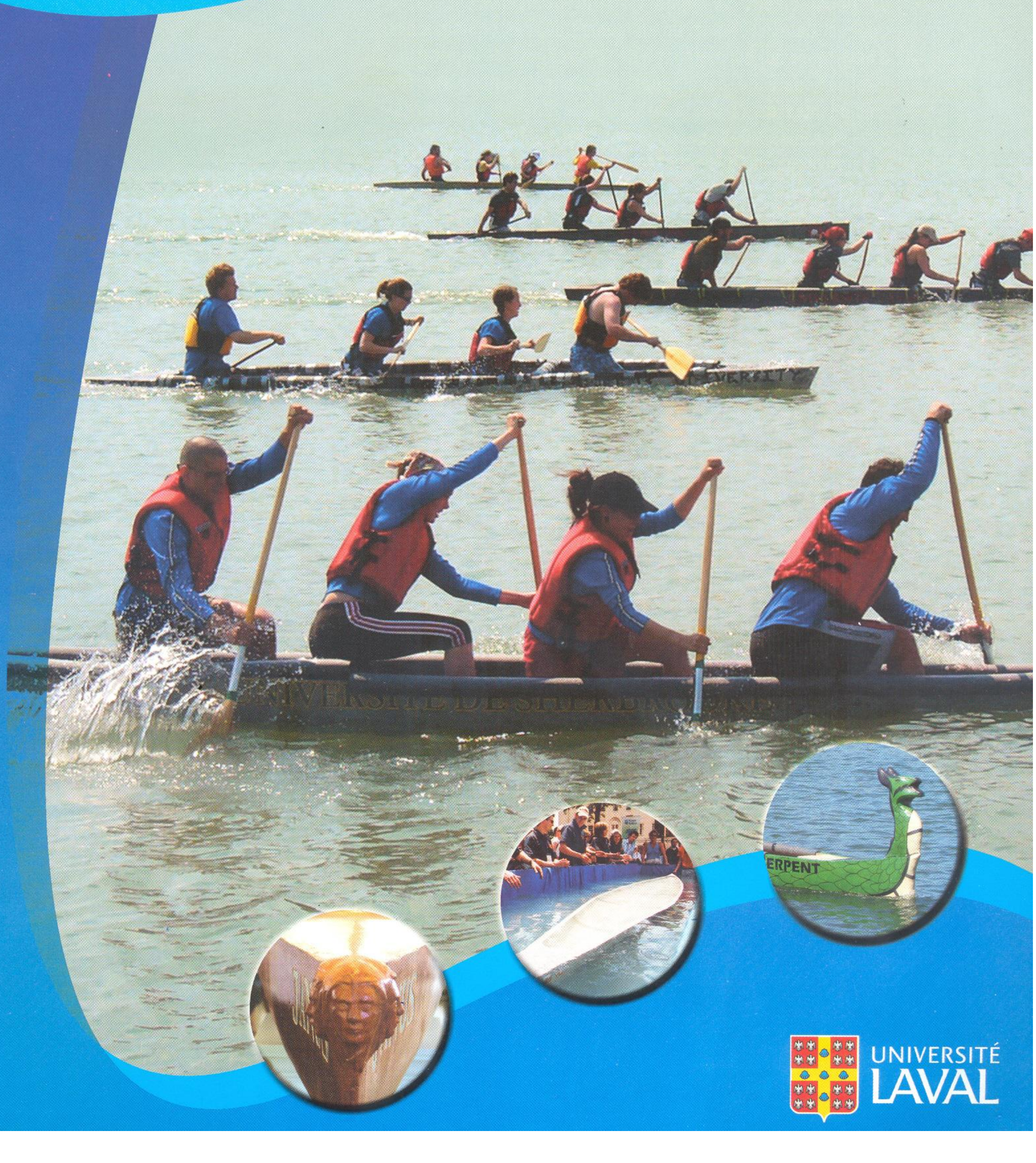

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MODAL ANALYSIS OF A LIGHTWEIGHT CONCRETE CANOE

By John A. Gilbert, Teng K. Ooi and Robert C. Engberg

ABSTRACT

This paper describes how experimental techniques and numerical methods were used to study the dynamic behavior (natural frequencies, mode shapes, and damping) of "Survivor," a concrete canoe that has become the benchmark for ongoing research in the area of strategically tuned absolutely resilient structures (STARS). This lightweight boat gave Team UAH the winning edge in the 2001 American Society of Civil Engineer's (ASCE) National Concrete Canoe Competition leading some researchers to speculate that cementitious materials may someday replace advanced aerospace composites.

Introduction

For the past eighteen years, concrete canoe teams competing in the American Society of Civil Engineer's (ASCE) National Concrete Canoe Competition have been studying the hydrodynamics, ergonomics, and structural performance of their boats' hulls. Students have continually been refining their boats' hull shapes using a combination of commercially available and customized software, and extensive experimental research has been performed to verify, refine, and fine-tune canoe models (Winsconsin-Madison 2006). These cumulative efforts have resulted in revolutionary developments in concrete technology not to mention an impressive collection of sleek and seaworthy concrete canoes.

The hull shapes of the winning entries at the national level are designed for different teams to race in different events and trade-offs must be made between speed, tracking, and maneuverability. Since all of the major competitors have similar goals, their boats look amazingly similar in shape (ConcreteCanoe.org 2005). But boats of similar shape and weight are not equally efficient because their dynamic response and modal parameters depend on the density, stiffness, and position of the materials employed during fabrication, as well as the physical constraints imposed by structural members and boundary conditions encountered while racing.

In this regard, it is important to realize that every structure tends to vibrate naturally and, when it does, shape changes occur. Also, while the structure is in service, external factors may produce excitations. Some frequencies affect the structure more than others and when one of the "natural" frequencies is reached, the structure resonates causing relatively large shape changes to occur. In modal analysis, the shape changes that correspond to these natural frequencies are referred to as vibration modes. The lowest natural frequency is called the "fundamental" frequency.

Avid canoeists that are fortunate enough to paddle a "good" boat often comment on the "rhythm" of their canoe. And, even though relations between the velocity and the modal response have yet to be quantified, understanding the dynamic behavior of a canoe's hull may hold the key to increasing the boat's average speed.

There is no question that a canoe's movement affects the boundary layer and wake, thereby creating perturbations in both skin and wave drag. And, although it will likely take a fairly sophisticated computational fluid dynamics model to fully quantify the latter, some teams have begun to hypothesize how modal parameters affect their canoe's performance.

In recent years, for example, Team UAH (UAH Team 2006) has been strategically tuning it's concrete canoes by lowering the natural frequencies of the hulls so the forcing function created by the paddlers (approximately 1 Hz) drives the boats toward resonance. The teams' designs are based on a new concept called "STARS" an acronym that stands for Strategically Tuned Absolutely Resilient Structures (Gilbert et al. 2005), and their approach represents a unique competition strategy.

When the flexible hulls deform in response to the torsional and bending moments applied, very large stresses and strains develop. The team strives to keep all of their materials elastic so the structure is absolutely resilient, enabling the strain energy stored in the deformed shape to be recovered. As they pull their paddles from the water, this energy is converted into forward propulsive momentum. The overall objective is to force the boat to surge forward between strokes so that it decelerates less, thereby allowing the team to achieve a higher average velocity.

The canoe that Team UAH built for the 2001 American Society of Civil Engineer's (ASCE) National Concrete Canoe Competition has become the benchmark for ongoing STARS research and this paper describes how experimental techniques and numerical methods were used to study the canoe's dynamic behavior.

Fabrication of *Survivor*

The structural design and analysis for *Survivor* were based on prior research conducted to determine the material properties of graphite reinforced cementitious composites (Biszick and Gilbert 1999, Vaughan and Gilbert 2001). The students relied on the large difference in stiffness between the constituents in their boat's composite section to drive the internal stress from a flexible cementitious matrix to three layers of relatively stiff reinforcement. They placed materials symmetrically to form an adaptive section optimized to resist stress reversals and strategically positioned fiber layers to tune the modal response.

Survivor was fabricated by placing the cementitious mixture described in Table 1 over three layers of graphite reinforcement. According to the manufacturer, each layer consisted of a non-impregnated graphite mesh with 3 000 fibers per tow, spaced at 3.18 mm (0.125 in) intervals. Each tow is 0.19 mm (0.0075 in) thick by 1.07 mm (0.042 in) wide; the elastic modulus and tensile strength of the graphite are 231 GPa (33.5 Msi) and 3.65 GPa (530 ksi), respectively.

The team began construction by coating a male mold with a thin sheet of plastic that served as a mold release. The first layer of graphite mesh [90°, 90°] was draped over the mold, and

Table 1. Mix proportions for 2001 *Survivor*

Component	Quantity (kg/m^3)
Portland Cement	266.2
Latex	51.7
Acrilic Fortifier	16.4
K25 Microspheres	104.3
Water	167.4

Figure 1. *Survivor* was tested in a free-hanging configuration

2.8 mm (0.11 in.) diameter speaker wires were positioned transversely at 7.6 cm (3 in.) intervals down the length of the hull.

The team prepared the cementitious mixture by initially mixing the cement and micro-spheres, and then added the acrylic fortifier, latex, and water to produce a mixture having a smooth texture. The team used drywall knives to level the mix to the upper surface of the wires. Once the mix had hardened, they removed the speaker wires and filled the grooves. This construction process was repeated for the second and third layers of graphite mesh.

Since the water required for hydration was held in the latex-modified system, the canoe was simply left to dry at room temperature. After only three days, the outer layer of concrete was hand-sanded smooth. The team filled voids with the same mix used during the main construction and then removed the canoe from the mold and repeated the process on the inner surface.

Using temporary wooden forms located around the upper rim of the canoe, the team placed a gunwale. Since the concrete canoe was inherently buoyant, no flotation was required.

Survivor is 6.8 m (22.3 ft) long and has a mass of 38.6 kg (equivalent to 85 lb), a maximum width of 81.3 cm (32 in.), and a maximum depth of 27.9 cm (11 in.) with no permanent stiffeners. The canoe's nominal wall thickness is 8.64 mm (0.34 in).

Experimental Testing

Standard impact hammer tests were conducted on the canoe to determine its modal parameters (natural frequencies, mode shapes, and damping). As illustrated in Figure 1, the boat was suspended using elastic cords in a free-hanging configuration.

The hull was struck using an impact hammer. A load cell, located at the tip of the hammer, was used to measure the force input while a tri-axial accelerometer was employed to measure the acceleration in G's along three perpendicular directions.

Frequency response functions (FRFs) were collected at different locations on the hull and the FRF data was combined by the Rational Fraction Polynomial method to determine the modal parameters. Each FRF represents the steady state transfer function of the dynamic system and describes the relation between the input and the output as a function of frequency in terms of gain and phase.

The model used to fit data comes from a linear, symmetric set of equations where the poles (frequency and damping) are defined in terms of global quantities and reciprocity is assumed to be inherent in the formulation of the equations (Avitabile 2000). The procedure is to first approximate how many modes there are in the bandwidth of interest and then estimate a pole for each mode. A residue is estimated for each mode. Mode shapes are curve fitted and computed based on these estimates using commercially available software.

As illustrated in Figure 2, a stability diagram is generated to show mode frequency and damping estimates. Commercially available software is applied to discriminate between stable, unstable, and noise modes. As the order of the model increases,

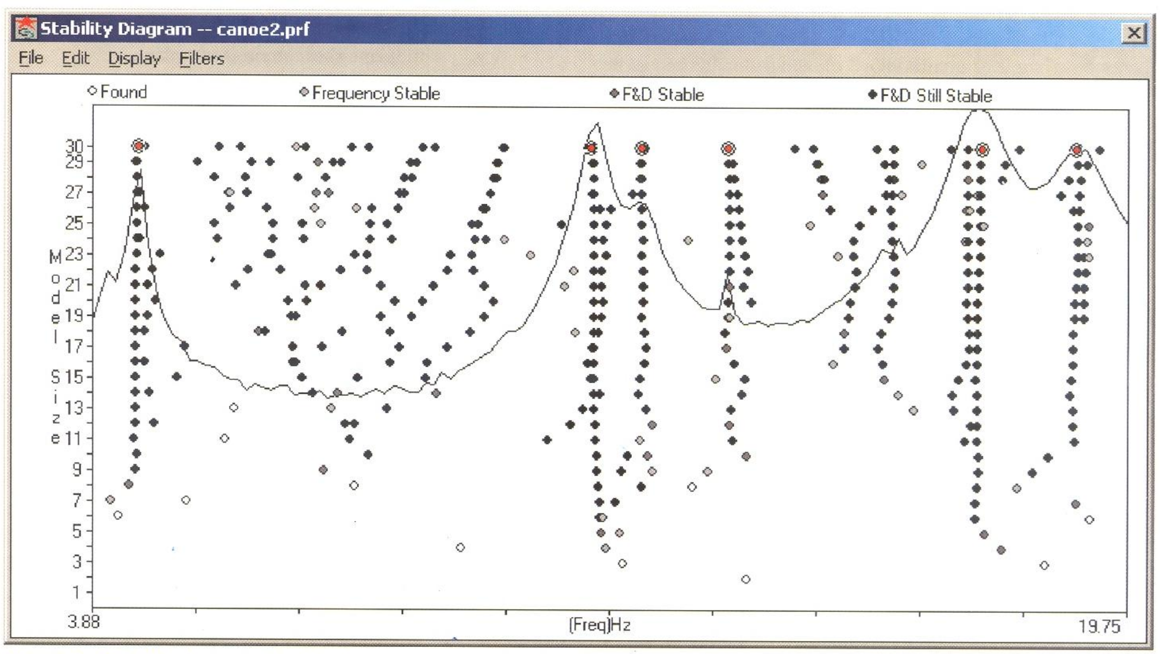


Figure 2. Stability diagram for *Survivor*

the poles are clearly identified. The red dots included in the figure, for example, correspond to predictions of the canoe's natural frequencies.

Table 2 shows data for the experimentally determined natural frequencies and percentage damping while Figure 3 shows wire frame plots of the first six canoe mode shapes. The corresponding natural frequencies are listed in the figure caption. The resolution is low, since only 35 points were sampled. But even though they are of low resolution, the plots provide some insight to the boats' dynamic responses.

The coherence transfer functions evaluated during the tests were close to 1.0 for most of the modes and the modes obtained after curve fitting all of the FRFs corresponded well with individual FRFs obtained throughout the structure. The largest off-diagonal value in the cross modal assurance criterion (MAC) matrix was 0.05. The average experimental damping for the first six modes was 1.96%.

Table 2. Natural frequencies and percentage damping

Mode Shape Number	2001 *Survivor*	
	Experiemental Fraquencies Hz	Experiemental Damping (%)
1	4.59	2.1
2	11.54	2.8
3	12.31	1.6
4	13.64	1.5
5	17.54	1.7
6	18.98	2.1

Numerical Analysis

A finite element analysis was also performed to evaluate the modal parameters. The finite element model was developed based on classical laminated plate theory using shell elements. The model was generated using dimensions from *Survivor* and material properties based on prior research (Vaughan and Gilbert 2001). MSC/NASTRAN® was selected as the finite element code because of its multi-layered composite element capabilities for normal mode analysis.

To be more specific, the constitutive equations that relate the force and moment resultants to the strains for graphite reinforced cementitious composite anisotropic plates can be derived based on the classical laminated plate theory and may be written in equation 1 (Kollar and Springer 2003).

The A, B, and D matrices are computed based on the laminate material properties, geometry, and stacking sequence. For a three-ply specially orthotropic laminate (laminate whose principal material axes are aligned with the natural body axes), these matrices were computed based on previously obtained

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{0y} \\ \gamma_{0xy} \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \quad \text{eq. 1}$$

where $(A_{ij}, B_{ij}, D_{ij}) = \sum_{r=1}^{N} \int_{z_r}^{z_{r+1}} Q_{ij}^{(r)}(1, z, z^2)dz$ and

N_x, N_y, N_{xy} = resultant force and shear force in x-axis, y-axis, and x-y plane, respectively

M_x, M_y, M_{xy} = resultant moment and twisting about x-axis, y-axis, and x-y plane, respectively

Z_r, Z_{r+1} = thickness coordinates of the lower and the upper surface of the r-th ply

$Q_{ij}^{(r)}$ = material stiffnesses of the r-th ply

Z = laminate transverse direction, normal to x-y plane

N = number of plies in the laminate

ε_{0x} = midplane strain in x-axis

ε_{0y} = midplane strain in y-axis

γ_{0xy} = midplane shear strain in x-y plane

K_x = plate bending curvature in the x-z plane

K_y = plate bending curvature in the y-z plane

K_{xy} = plate twisting curvature in the x-y plane

A_{ij} = extensional stiffnesses

B_{ij} = bending-extension coupling stiffnesses

D_{ij} = bending stiffnesses

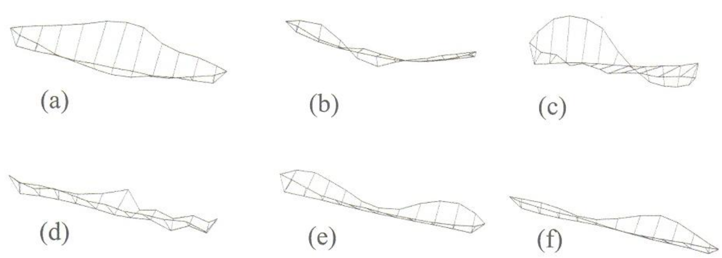


Figure 3. Impact hammer test results showing the first six modes of *Survivor*, (a) 1st mode, (b) 2nd mode, (c) 3rd mode, (d) 4th mode, (e) 5th mode, (f) 6th mode

material properties (Vaughan and Gilbert 2001). In this configuration, the bending-extensional coupling coefficients, B_{ij}, the bending-twisting coefficients, D_{13}, D_{23}, and the shear-extensional coupling coefficients, A_{13}, A_{23}, are all zero.

The material properties are 3.48 GPa (0.51 Msi) for the elastic modulus, 517 MPa (74.98 ksi) for the shear modulus, 0.137 for the Poisson's ratio, and 757 kg/m^3 (47.30 lb/ft^3) for the mass density. Each ply is assumed to have a thickness of 2.54 mm (0.10 in).

The finite element model consisted of quadrilateral membrane-bending plate elements with uniform thickness. Although transverse shear stresses are necessary for force equilibrium under transverse loads, they were neglected because they vanish on the free surfaces and are small in magnitude compared to the in-plane stresses. The transverse normal stress was also neglected because it is small compared to the in-plane normal stresses.

Damping was neglected in the analysis and refinements in element sizes were made until convergence was achieved with regard to the natural frequencies. Only the lower natural frequencies and their associated mode shapes in a free-free boundary condition were calculated because they are sufficient to adequately describe the dynamic behavior of the canoe.

Table 3. Comparisons of finite element predictions with modal test results for *Survivor*

Mode Shape Number	Finite Element Frequencies Hz	Experimental Frequencies Hz	Relative Difference Hz
1	7.31	4.59	2.72
2	10.76	11.54	0.78
3	12.06	12.31	0.24
4	15.38	13.64	1.74
5	15.95	17.54	1.59
6	18.52	18.98	0.46

Table 3 shows data for the natural frequencies obtained for *Survivor* and compares the finite element predictions of the natural frequencies with the experimentally measured values. With the exception of the frequencies obtained for the first mode, the results agree quite well.

Figure 4 shows exaggerated plots of the first six mode shapes for *Survivor* predicted by finite element analysis. The corresponding natural frequencies are listed in the figure caption.

The finite element analysis shows the first mode shape corresponds to an anti-symmetrical torsional deformation where the center of the canoe remains stationary and the sides of the canoe open. The second mode is a bending mode in which the canoe flutters like a butterfly. The other higher modes are much more complex. Even though the number of points at which experimental measurements were made is small, the experimental mode shapes (see Figure 3), with the exception of the first mode, are representative of those predicted by the finite element analysis (see Figure 4). The close match validates the finite element approach, implying that the model can be used to accurately predict the dynamic response of other large cementitious structures.

Discussion

The differences in the frequency and mode shape between the measured and calculated values for the first mode are partially attributed to difficulties in detecting low frequencies with the available instrumentation and the relatively small number of points sampled during the modal tests. Smaller discrepancies between the measured and calculated frequencies can be partially attributed to two problems in signal processing. The first is the noise present in the force or response signal as a result of a long time record. The second is the leakage present in the response signal as a result of a short time record.

Other reasons for frequency differences are that impact signals may be poorly suited for the frequency response function measurements; resolution bias errors may be present in the spectral estimates; and, the system relating the output and input may not be linear. The main difference between the measured and calculated frequencies is most likely due to aberrations encountered while experimentally determining the material properties and the subsequent inaccuracies introduced into the constitutive equations used in the finite element analysis.

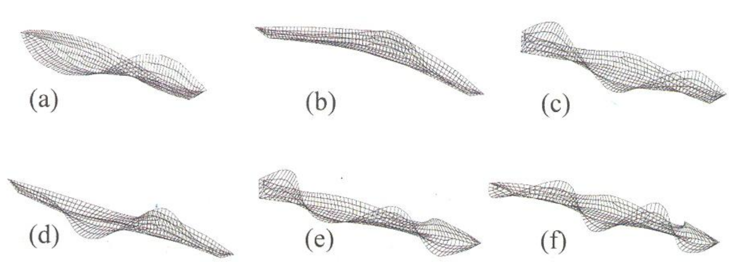


Figure 4. Finite element results showing the first six modes of *Survivor*, (a) 1st mode, (b) 2nd mode, (c) 3rd mode, (d) 4th mode, (e) 5th mode, (f) 6th mode

Conclusion

An agreement between the experimental modal parameters and the finite element predictions described above indicates that standard impact modal testing can be applied to test concrete canoes made of cementitious materials that have been characterized by the classical laminated plate theory.

Results indicate that boats of similar shape and weight are not equally efficient because their dynamic response and modal parameters depend on the density and stiffness of the materials, the fiber orientation and stack sequence employed during fabrication, and the physical constraints imposed by structural members and boundary conditions encountered while racing. And, even though relations between the velocity and the modal response have yet to be quantified, understanding the dynamic behavior of a canoe's hull may hold the key to increasing its average speed.

During the past two years, for example, Team UAH reported producing concrete canoes that provided a 12% increase in speed over the speeds recorded by the same teams in fiberglass prototypes that had the same dimensions and weight. In addition to being made from different materials, these prototypes included thwarts and had a totally different modal response.

As hull speeds increase and finishing times get closer, teams must look more closely at all aspects of their designs to obtain a winning edge over their competitors. And, it should be clear from the results presented above that it is not necessarily the physical characteristics of the hull that will determine the winner.

Acknowledgements

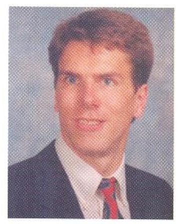
The authors would like to acknowledge all of the faculty, students, staff, supporters, and sponsors that make Team UAH's participation in the National Concrete Canoe Competition possible. Dr. Houssam Toutanji currently serves as co-faculty advisor to the chapter with Dr. Gilbert.

As of 2005, Team UAH has proudly represented the Southeast Region thirteen times at the national level in the ASCE (American Society of Civil Engineers) National Concrete Canoe Competition and they have five national titles and three, second place finishes to their credit.

Biographies

Dr. John A. Gilbert is a professor at the University of Alabama in Huntsville where he served as co-faculty advisor for the school's ASCE Student Chapter for the past twenty years. He specializes in engineering mechanics and has been the recipient of numerous awards at UAH including the Alumni Association's Distinguished Faculty Award, the Foundation's Distinguished Teaching Award, the College of Engineering's Outstanding Faculty Member Award, and the Student Government Outstanding Faculty Award. John has written many articles on concrete canoeing and currently serves as the web master for concretecanoe.org.

Dr. Teng K. Ooi received his Ph.D. in Mechanical Engineering from UAH in 2002 and has served as an outside advisor to the ASCE Student Chapter there since graduation. He specializes in numerical analysis of composite materials and works as the Science and Technology Lead Engineer for the US Army in the Ground Based Midcourse Defense, Ground Based Interceptor, Exoatmospheric Kill Vehicle Product Office located in Research Park, Huntsville, AL. Teng holds academic positions at UAH and Stanford University and is a visiting scholar at UCLA.



Mr. Robert C. Engberg received his M.S.E. degree in Mechanical Engineering from UAH in 2004. He was an active member of the ASCE Student Chapter while at UAH and currently assists students there with experimental testing. He specializes in experimental modal analysis of composite materials and works as an aerospace engineer in the Structural and Environmental Test Division at NASA's Marshall Space Flight Center in Huntsville, Alabama..

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