Digital Image Correlation of Stereoscopic Images for Radial Metrology

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ABSTRACT

This paper describes a computational technique that can help locate points on the surfaces of objects surrounding a panoramic imaging system. Stereoscopic measurements are made by digitally recording two annular images captured before and after a panoramic annular lens is translated along its optical axis. The annular images are linearized by rolling segments along their outer circumference, and moving all the pixels between the contact points and the center of the image to vertical lines via a mapping function. The segments transform into rectangles in which circumferential and radial lines appear horizontal and vertical, respectively. Each column in the linearized segment corresponds to a different radial position angle while each pixel location in the column corresponds to a different field angle. These relationships are quantified and the linearized images are then compared and analyzed by the method of digital correlation.

INTRODUCTION

The authors previously developed a stereoscopic system for radial metrology to pinpoint the locations and sizes of objects contained within the field of view of a panoramic imaging system [1]. The stereoscopic system included a panoramic annular lens (PAL) [2] and measurements were made by digitally recording two annular images captured before and after the PAL was translated along its optical axis. The images of objects contained within the field of view moved radially as the lens was translated. Since the pixel shift was related to the distance between the object and the initial position of the PAL, and since the radial position angle of a point was known from its pixel location in the image(s), the position coordinates were uniquely determined.

The stereoscopic system was calibrated to relate pixel locations in the annular images to field angles so that measurements could be made. But distortions associated with mapping objects surrounding the lens to the image plane made it difficult to recognize and pinpoint shifted images of the same object without human intervention.

When the lens is translated, objects move radially and their aspect ratios remain constant. But the problem is that their sizes do not. This paper describes one method for solving this problem by combining the methods of image linearization and digital correlation.

PANORAMIC ANNULAR LENS

A schematic diagram for a panoramic imaging system is shown in Fig. 1. The PAL, shown at the top of the figure, forms an internal virtual image of its surroundings by a combination of reflection and refraction. A collector lens is employed to produce an inverted, flat annular image.

As illustrated in the figure, the optical axis of the PAL is defined by a line perpendicular to the rear flat surface, which passes through the centers of curvature of its three spherical surfaces. A longitudinal axis, labeled Z, is chosen to coincide with the optical axis. Two other axes, labeled X and Y, are established in a plane defined by the physical equator of the lens. They are chosen to form a right-handed triad with the longitudinal axis. Cylindrical \((r,\theta,z)\) coordinates may also be defined with respect to the origin in real space. The angle \(\theta\), measured counterclockwise from X, is called the radial position angle.

At a given \(\theta\), all rays in the object space intersect at a common point called the entrance pupil. Point \(O_e\) on Fig. 1, for example, corresponds to \(\theta = 90^\circ\). A field angle, \(\phi\), can be included as one of three spherical coordinates \((\rho,\phi,\theta)\) measured from a local system situated at this point. For \(B/2 \geq \theta \geq -B/2\) and \(2\pi \geq \theta \geq 0\), the position coordinates measured relative to the Cartesian coordinate system are

\[
x = (o_p + \rho \cos \phi) \cos \theta \\
y = (o_p + \rho \cos \phi) \sin \theta \\
z = h_p + \rho \sin \phi
\]
where \( o_p = 1.342 \text{ mm (0.053 in.)} \) and \( h_p = 6.740 \text{ mm (0.265 in.)} \) for the 38.9 mm (1.53 in.) diameter PAL used in the present study.

Figure 1. A panoramic imaging system based on a panoramic annular lens (PAL).

Referring again to Fig. 1, the image space is defined by either Cartesian \((x',y')\) or polar \((r',\theta')\) coordinates measured from an origin situated at the center of the annulus. Points located on the inner radius, at a radial distance of \( r_i' \) in the image plane, correspond to objects viewed at the maximum field angle; points located on the outer radius at \( r_o' \), correspond to objects viewed at the minimum field angle.

In practice, the camera system is mounted vertically on a tripod with the mounting surface aligned along the X-axis; the Y-axis is directed away from the base. An observer looking from behind the camera in the Z direction would see the X-axis to their left with Y upward. From this perspective, the radial position angle, \( \theta \), is measured clockwise from the X-axis. When the image is viewed on a monitor, the X’-axis is directed toward the right with Y’ downward. The radial position angle, \( \theta' \), is measured clockwise from the X’-axis.

STEREOSCOPIC ANALYSIS

Figure 2 shows a schematic of the \( r,z \) plane of a cylindrical coordinate system with point \( P \) located in real space at a fixed radial position angle, \( \theta \). The origin of the global coordinate system is chosen at the center of the PAL initially located at point \( O_1 \). A local spherical cylindrical coordinate system is defined at the entrance pupil of the lens at point \( O_{1e} \). The field angle measured to the point in question is \( \phi_{p1} \).

When the lens is moved downward through a distance, \( \Delta \), to point \( O_2 \), point \( P \) is observed from \( O_{2e} \) at angle, \( \phi_{p2} \). The distance \( O_{1e}P \) corresponds to the position vector in the local spherical coordinate system, and from trigonometry,
\[ \rho = \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \]  

(2)

\[ \phi = 0.0014 (r_1')^2 - 0.8839 r_1' + 105.73 \]  

(3)

The Cartesian coordinates of the point are found by substituting Eq. (2) into Eq. (1) with \( \phi = \phi_1 \) as,

\[
\begin{align*}
    x &= o_x + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \cos \theta \\
    y &= o_y + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \sin \theta \\
    z &= h + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \sin \phi_1
\end{align*}
\]

(4)

where \( o_x = 1.342 \text{ mm (0.053 in.)} \) and \( h = 6.740 \text{ mm (0.265 in.)} \).

Thus, for a given \( \Delta \), the location of the object can be found by first measuring the radial position angle \( \theta' \), in the image plane. Since \( \theta' = \theta \), and Eq. (3) can be used to convert the image distances \( r_1' \) and \( r_2' \) to \( \phi_1 \) and \( \phi_2 \), respectively, Eq. (4) can be applied.
A procedure was previously developed to linearize segments of the annular images acquired from a PAL [4]. Figure 3, for example, shows a reconstruction of the digital image acquired and stored when the image system depicted in Fig. 1 is positioned along the axis of a cylindrical pipe, the interior surface of which is covered with a test pattern. The test pattern contains a different pattern, i.e., diamonds, squares, checkerboard, and concentric circles, in each quadrant of the cavity wall. The image was photographed directly from a VGA monitor; the insert corresponds to a linearized version of the fourth quadrant produced by using digital image processing.

![Image](image1.png)

**Figure 3.** A linearized segment superimposed on a test image acquired using the system shown in Figure 1.

Two stages of linearization are needed: (1) tangential linearization and (2) radial linearization. In the tangential linearization, a wedge-shaped portion of the annular image of the inside of the pipe is converted into a rectangular section. ‘Rolling’ the annular image along its outer circumference and moving all the pixels between the contact point and the center of the image to an appropriate location on a vertical line in the final rectangular image accomplish this. Next, because the annular image is not linear in the radial direction, a vertical stretching of the rectangular image is required; this second process is radial linearization.

A normalized calibration curve was later generated for points lying between the inner and outer radius of the annual image [6] and, as illustrated in Figure 4, this transformation can be used to linearize the entire image.

![Image](image2.png)

**Figure 4.** The entire annular image shown in Figure 3 can be linearized.

**DIGITAL CORRELATION**

Digital correlation is a method of pattern recognition in which a small subset of an initial image is located in a second displaced image. A standard equation used to determine the correlation coefficient, c, for a window centered over location (m,n) in the displaced image is, [3]
\[
c(m,n) = \frac{\sum_x \sum_y [f(x,y) - \langle f \rangle] [w(x-m,y-n) - \langle w \rangle]}{\left[ \sum_x \sum_y [f(x,y) - \langle f \rangle]^2 \sum_x \sum_y [w(x-m,y-n) - \langle w \rangle]^2 \right]^{1/2}}
\]

where \(f(x,y)\) are the intensity values of the second image for those locations under the window values \(w(x,y)\) as the window is moved over a suitably chosen search range \((x,y)\); \(\langle f \rangle\) is the average intensity value of the region located under the window and \(\langle w \rangle\) is the average intensity value of the window. The maximum correlation value indicates the best match of the chosen subset from the initial image as located in the second image.

**EXPERIMENTAL**

A simulation was conducted where it was assumed that a section of a 0.91 m (3 ft) diameter cylindrical pipe was to be studied as part of an on-line sewer inspection project. To this end, a flat test card was designed so that when it was positioned in the field of view of the stereoscopic measurement system, the offsets to the seven targets drawn on the card were contained in the range 30.48 cm (12 in.) \(< y < 45.72 \text{ cm (18 in.)}. In addition, the spacing between the targets was adjusted so that the offset increased by 2.54 cm (1 in.) increments.

A stereoscopic system was built by mounting a PAL imaging system on a Pulnix CCD camera. The lens was oriented with its optical axis in the vertical direction and the camera was mounted on a tripod that allowed it to be moved downward. Images were stored as 512 x 512 pixel arrays, having 256 intensity levels, using a Matrox MVP-AT frame grabber and IPPLUS software. Figure 4 shows a photograph taken while conducting the simulation.

![Figure 4. Photograph showing the PAL and the test card.](image)

Figure 5, on the other hand, shows a schematic diagram of the test card as if it were placed within a 0.91 m (3 ft) diameter cylindrical pipe. For practical purposes, the upper surface of the card could represent a sludge deposit in the bottom of the pipe. The configuration depicted in the figure corresponds to the case where the camera is fed into the pipe “right side up.” The axes are labeled as viewed by an observer looking into the pipe from behind the camera.

The point labeled as “1” has an offset of 30.48 cm (12 in.) and is positioned at a radial position angle, \(\theta = 270\) degrees. Point “7”, on the other hand, has an offset of 45.72 cm (18 in.) with \(\theta = 221.8\) degrees.
The annular images shown in Figure 6 correspond to a test conducted with a shift of 14.9 cm (5.9 in.). In this case, the card was positioned so that all targets were initially at a distance of -9.59 cm (–3.78 in.) below the equator of the lens ($\phi_1 = -18.7$ degrees for Point No. 1).

Figure 5. Target location for pipe simulation.

Figure 6. PAL images of simulated targets.

Figure 7 shows the linearized images. The width of the image was divided into 360 columns corresponding to one-degree increments in the radial field angle.
The method of digital correlation was applied to the images in Fig. 7. Since all points move radially in the annular image, only a one-dimensional (vertical) search was required.

The table included as Fig. 8 shows the offset and radial position angles of the targets along with position vectors measured by the system. The latter are compared to those computed from trigonometry.

<table>
<thead>
<tr>
<th>Pt</th>
<th>Offset (cm)</th>
<th>( \theta ) (deg)</th>
<th>( \rho_{\text{sys}} ) (cm)</th>
<th>( \rho_{\text{real}} ) (cm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.48</td>
<td>270.0</td>
<td>32.23</td>
<td>32.03</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>33.02</td>
<td>247.4</td>
<td>34.01</td>
<td>34.44</td>
<td>-1.28</td>
</tr>
<tr>
<td>3</td>
<td>35.56</td>
<td>239.0</td>
<td>37.34</td>
<td>36.88</td>
<td>1.21</td>
</tr>
<tr>
<td>4</td>
<td>38.10</td>
<td>233.1</td>
<td>39.29</td>
<td>39.32</td>
<td>-0.09</td>
</tr>
<tr>
<td>5</td>
<td>40.64</td>
<td>228.6</td>
<td>43.48</td>
<td>41.78</td>
<td>4.04</td>
</tr>
<tr>
<td>6</td>
<td>43.18</td>
<td>224.9</td>
<td>45.39</td>
<td>44.25</td>
<td>2.60</td>
</tr>
<tr>
<td>7</td>
<td>45.72</td>
<td>221.8</td>
<td>46.30</td>
<td>46.74</td>
<td>-0.90</td>
</tr>
</tbody>
</table>

There was some scatter in the calibration data taken that corresponded to differences in field angles on the order of ± 1.5 degrees. The scatter leads to errors, since position measurements depend upon the field angle. In general, the errors increase for smaller shifts and larger radial offset distances. When the shift is small and the offset large, the stereoscopic system may become unstable, forcing the position vector to infinity and beyond (negative). Consequently, system design is of critical importance [1].

The ultimate performance of the system depends upon the resolution of the digitized image. Assuming that a target must fall within a given pixel and, in the most severe case, measurements are made along the diagonal (at \( \theta' = 45^\circ, 135^\circ, 225^\circ, 315^\circ \)), the best resolution that can be achieved without using interpixel interpolation is 0.7 pixels. Noting that the highest slope on the calibration curve (0.6 degrees/pixel) occurs at the largest field angle [1]; the smallest scatter that can be achieved over the entire field of view is ± 0.4 degrees. This corresponds to about 6 percent error.

**CONCLUSIONS**

This work demonstrated how the method of digital correlation could be applied to stereoscopic images recorded using a panoramic imaging system. A pipe simulation was used to illustrate the approach and targets were located within five percent error.
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REFERENCES


