Contour Measurement using Radial Metrology

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ABSTRACT

This paper describes techniques developed to locate points on the surfaces of objects surrounding a panoramic imaging system. These techniques range from profiling surfaces using structured lighting and the moiré method to employing techniques such as holographic interferometry and digital image correlation for analyzing speckle patterns and stereoscopic images.

Topics to be discussed include the optical characterization of the lens, the polar mapping function associated with the PAL, and selected tests conducted to determine and verify the critical parameters required for contouring. In many cases, annular images are linearized by rolling segments along their outer circumference, and moving all the pixels between the contact points and the center of the image to vertical lines via the mapping function. Examples are included to demonstrate this approach.

INTRODUCTION

The invention of the panoramic annular lens (PAL) [1] led to the science of radial metrology [2]: the process of using panoramic imaging systems for inspection and measurement. During many of the inspection and measurement tasks associated with radial metrology [3], it is desirable to know the exact locations and sizes of objects contained within the field of view. Contour information is also important especially when displacement, strain, and stress must be transformed to obtain critical parameters such as principal stress or maximum shear.

PANORAMIC ANNULAR LENS

A schematic diagram for a panoramic imaging system is shown in Fig. 1. The PAL, shown at the top of the figure, forms an internal virtual image of its surroundings by a combination of reflection and refraction. A collector lens is employed to produce an inverted, flat annular image.

As illustrated in the figure, the optical axis of the PAL is defined by a line perpendicular to the rear flat surface, which passes through the centers of curvature of its three spherical surfaces. A longitudinal axis, labeled Z, is chosen to coincide with the optical axis. Two other axes, labeled X and Y, are established in a plane defined by the physical equator of the lens. They are chosen to form a right-handed triad with the longitudinal axis. Cylindrical (r,θ,z) coordinates may also be defined with respect to the origin in real space. The angle θ, measured counterclockwise from X, is called the radial position angle.
At a given $\theta$, all rays in the object space intersect at a common point called the entrance pupil. Point $O_e$ on Fig. 1, for example, corresponds to $\theta = 90^\circ$. A field angle, $\phi$, can be included as one of three spherical coordinates $(\rho, \phi, \theta)$ measured from a local system situated at this point. For $\pi/2 \geq \theta \geq -\pi/2$ and $2\pi \geq \theta \geq 0$, the position coordinates measured relative to the Cartesian coordinate system are

$$
\begin{align*}
x &= (o_p + \rho \cos \phi) \cos \theta \\
y &= (o_p + \rho \cos \phi) \sin \theta \\
z &= h_p + \rho \sin \phi
\end{align*}
$$

where $o_p = 1.342$ mm (0.053 in.) and $h_p = 6.740$ mm (0.265 in.) for the 38.9 mm (1.53 in.) diameter PAL used in the present study.

Referring again to Fig. 1, the image space is defined by either Cartesian $(x',y')$ or polar $(r',\theta')$ coordinates measured from an origin situated at the center of the annulus. Points located on the inner radius, at a radial distance of $r_i'$ in the image plane, correspond to objects viewed at the maximum field angle; points located on the outer radius at $r_o'$, correspond to objects viewed at the minimum field angle.

In practice, the camera system is typically mounted vertically on a tripod with the mounting surface in the X-Y plane with the Y-axis directed away from the base. An observer looking from behind the camera in the Z direction would see the X-axis to his/her left with Y upward. From this perspective, the radial position angle, $\theta$, is

![Figure 1. A panoramic imaging system based on a panoramic annular lens (PAL).](image-url)
measured clockwise from the X-axis. When the image is viewed on a monitor, the X'-axis is directed toward the right with Y' downward. The radial position angle, θ', is measured clockwise from the X'-axis.

In addition to imaging its surroundings, a PAL can be used to illuminate them. Figure 2, for example, shows a 38 mm (1.5 in.) diameter PAL with its optical axis aligned with the longitudinal axis of a pipe having an inner radius equal to R. The characteristics of the lens were determined by combining physical measurements with a ray trace. The angular field of view measured from the entrance pupil is 45.4° covering the range -18.8° ≤ φ ≤ 26.6°, where φ is the field angle. By convention, positive field angles are measured toward the front of the lens.

![Figure 2. Optical characteristics of an illuminating lens.](image)

The linear distance that can be illuminated or imaged by the lens is defined in Fig. 2 as the field of view (FOV). For the 38 mm (1.5 in.) diameter PAL,

\[
\text{FOV} = (R - y_i)(\tan 26.6° + \tan 18.8°).
\]

Not all the rays exiting or entering the lens actually pass through the entrance pupil center, so a linear regression was performed on ray trace data to establish a parametric equation describing the direction of propagation of the illuminating wavefront, given by the angle γ, in terms of the field angle, φ [4]. For a plane wavefront entering the flat surface of the PAL,

\[
\gamma_i = 1.00417 \phi_i - 0.2978
\]

where both angles are measured in degrees. Over the FOV, the propagation directions determined from this equation agree to within ± 2% error with those obtained from the ray trace.

The 38 mm (1.5 in.) diameter PAL has been characterized in terms of spherical aberration and coma, distortion, image plane curvature, and the modulation transfer function [5]. In general, the acceptance angle varies with the field angle; the amount of spherical aberration is proportional to the acceptance angle. The magnification varies quadratically and image plane curvature is cubic.

From an experimental mechanics standpoint, the resolution of the PAL varies from the forward viewing edge to the back viewing edge with an angular resolution of approximately 6 millirads. Even though the PAL is not strictly
afocal, objects appear to be in focus from the lens surface to infinity. The transmittance varies less than five percent over the visible light range; however, since the PAL is both refractive and reflective, it does not possess the same performance for all wavelengths.

**MAPPING CHARACTERISTICS**

In radial metrology, the aspect ratio of an area is defined as height divided by width [6]. In real (or object) space, height is measured as the longitudinal distance relative to the optical axis of a lens; width corresponds to the circumferential distance measured around the optical axis. In image space, height is measured as a radial distance relative to the center of an image; width corresponds to the medial circumferential distance of each element measured around the image center.

When a conventional lens is used to image the inside wall of a cylinder whose inside surface is composed of a uniform grid of squares, the structure is mapped into the image plane as a series of evenly spaced concentric rings representing equally spaced lines drawn around the circumference of the cylinder. Radial lines represent the longitudinal lines drawn along the length of the cylinder at constant circumferential positions. Figure 3(a) illustrates that, in the case of the conventional lens, square elements having a real space aspect ratio of unity are mapped to an image comprised of segments that have different image plane aspect ratios. The PAL maps the same uniform grid of squares into the constant aspect ratio polar map illustrated in Figure 3(b). This unique property results in a higher information density thereby facilitating cavity inspection and measurement [7].

![Figure 3. A square grid wrapped around the inside wall of a cylinder becomes: a conventional polar map (a) when recorded with a conventional lens, and a constant aspect ratio polar map (b) when recorded with a PAL.](image)

**IMAGE LINEARIZATION**

A procedure has been developed to linearize segments of the annular images acquired from a PAL [8]. Figure 4, for example, shows a reconstruction of the digital image acquired and stored when the image system depicted in Fig. 1 is positioned along the axis of a cylindrical pipe (see Fig 2), the interior surface of which is covered with a test pattern. The test pattern contains a different pattern, i.e., diamonds, squares, checkerboard, and concentric circles, in each quadrant of the cavity wall. The image was photographed directly from a VGA monitor; the insert corresponds to a linearized version of the fourth quadrant produced by using digital image processing.

Two stages of linearization are needed: (1) tangential linearization and (2) radial linearization. In the tangential linearization, a wedge-shaped portion of the annular image of the inside of the pipe is converted into a rectangular section. ‘Rolling’ the annular image along its outer circumference and moving all the pixels between the contact point and the center of the image to an appropriate location on a vertical line in the final rectangular image accomplish this. Next, because the annular image is not linear in the radial direction, a vertical stretching of the rectangular image is required; this second process is radial linearization.
Figure 4. A linearized segment superimposed on a test image acquired using the system shown in Figure 1.

A normalized calibration curve was later generated for points lying between the inner and outer radius of the annual image [9] and, as illustrated in Figure 5, this transformation can be used to linearize the entire image.

Figure 5. The entire annular image shown in Figure 4 can be linearized.

CONTOUR MEASUREMENTS

The wide field of view of the PAL makes it possible to construct systems that can view and contour salient features relative to their surroundings. This may be important in cases where chemical deposits cause corrosion, or where combinations of thermal and mechanical stresses cause wear or produce cracks. Such conditions are typically encountered in nuclear power plants and in rocket engines where many components, designed to function at high temperatures and pressures, must be periodically inspected to avoid catastrophic failures. Figure 6, for example, shows a 7.62cm (3.0in) diameter Panoramic Video System (PVS) prepared as a deliverable under a NASA contract to inspect the Space Shuttle Main Engine (SSME).

The PVS relies on a PAL {1} to capture a cylindrical view of the region surrounding the lens through a transparent window {2}. Incandescent illumination is distributed over the cylindrical field of view using a light ring {3} and an optical waveguide {4}. Projecting structured light into the field of view using a laser diode {5} and a rotating mirror driven by a motor {6} provides measurement capabilities. Panoramic images are acquired with a digitizing camera {7} and stored in a modified image enhancer. The enhancer includes menu driven image processing software to linearize the annular images and make measurements within cavities.
Figure 6. A Panoramic Video System (PVS): 1. PAL, 2. transparent viewing window, 3. incandescent light, 4. cylindrical waveguide, 5. laser diode, 6. rotating mirror and motor, 7. digitizing camera.

When measurements are made using structured light, linearization of the annular image may be beneficial. Figure 7, for example, shows the results obtained from a PVS designed to visually detect inclusions located on the inner wall of a pipe. In this case, a laser diode and a rotating mirror produce a scan, which was originally circular, that traces out shapes in the annular image which are "similar" to those of the inclusions.

Figure 7. PAL image showing a laser scan over inclusions located on the wall of a cylindrical pipe.

The interpretation becomes clear when the image is linearized. Figure 8, for example, shows a linearized version of the lower portion of the annular image. The bottom of the trace represents a constant radial distance from the optical axis of the PAL to the wall of the pipe. The shape and dimensions of the inclusions can be easily observed and measured with respect to this baseline using a relatively simple edge finding algorithm. The optical system can be designed so that measurements are based on a linear calibration curve.

As opposed to using a single line trace, it is possible to record moiré fringes corresponding to displacement fields obtained by combining inscribed or projected gratings [10]. Full-field displacement patterns can also be recorded using the method of speckle photography and digital correlation [11].
PHASE MEASUREMENT

A pair of lenses can be used to make phase measurements [12]. In this case, one PAL is used to illuminate the cavity while the other is used to view the inner wall. Figure 9, for example, shows a setup used to record holographic interferograms. A cylindrical stop was positioned between the panoramic lenses to block stray light from entering the observation lens. Image quality was enhanced by mounting an annular shaped mask on a glass plate located in the image plane of the transfer lens (component 11). The object and reference beams were both vertically polarized to maximize interference at the thermoplastic plate. Data Translation video capture software was employed to record the holographic information. The filter operations and contrast adjustments provided by the software were used to enhance image quality.

Figure 8. The lower portion of the image shown in Figure 7 is linearized to facilitate optical measurement.


Figure 10a, for example, shows a holographic interferogram obtained when a diametral load is applied to a section of pipe. The interpretation of this pattern requires the application of standard interferometric analysis techniques such as phase shifting and phase stepping methods, heterodyne holographic interferometry or carrier fringe methods [13, 14]. Since the distortions inherent in the PAL image make this process difficult, they must be removed by the linearization procedure described earlier.

Electronic speckle pattern interferograms can also be captured by making slight modifications in the optical system [15]. Figure 10b, for example, shows a noisy unfiltered ESPI image obtained when a diametral load is applied to a section of pipe.
Figure 10. Interferometric fringe patterns obtained by applying a vertical compression to a pipe: (a) panoramic holointerferogram, and (b) electronic speckle pattern interferogram.

The optical setup shown in Fig. 9 also allows time average holo-interferograms to be recorded [16]. Figure 11, for example, shows the modal response of a liner used in a turbopump designed for use in the Space Shuttle Main Engine (SSME).

Figure 11. A time average holointerferogram recorded on the inner surface of a turbopump liner.
STEREOSCOPIC ANALYSIS

A stereoscopic system was developed for radial metrology to pinpoint the locations and sizes of objects contained within the field of view of a panoramic imaging system [17]. The stereoscopic system included a panoramic annular lens (PAL) and measurements were made by digitally recording two annular images captured before and after the PAL was translated along its optical axis. The images of objects contained within the field of view moved radially as the lens was translated. Since the pixel shift was related to the distance between the object and the initial position of the PAL, and since the radial position angle of a point was known from its pixel location in the image(s), the position coordinates were uniquely determined as described below.

Figure 12 shows a schematic of the $r,z$ plane of a cylindrical coordinate system with point $P$ located in real space at a fixed radial position angle, $\theta$. The origin of the global coordinate system is chosen at the center of the PAL initially located at point $O_1$. A local spherical cylindrical coordinate system is defined at the entrance pupil of the lens at point $O_{1e}$. The field angle measured to the point in question is $\phi_{p1}$.

When the lens is moved downward through a distance, $\Delta$, to point $O_2$, point $P$ is observed from $O_{2e}$ at angle, $\phi_{p2}$. The distance $O_{1e}P$ corresponds to the position vector in the local spherical coordinate system, and from trigonometry,

$$\rho = \frac{\Delta \cos \phi_{p2}}{\sin (\phi_{p2} - \phi_{p1})}$$

\[4\]

Figure 12. Stereoscopic images are recorded with the PAL in two different locations.

Since the radial position angle is the same for both PAL recordings, the point $P$ moves along a radial line in the image from $r'_{p1}$ to $r'_{p2}$. The system was calibrated [17] and each of these radial coordinates corresponds to a field angle, $\phi$, measured in degrees and given by the relation

$$\phi = 0.0014 (r'_p)^2 - 0.8839 r'_p - 105.73$$

\[5\]
The Cartesian coordinates of the point are found by substituting Eq. (4) into Eq. (1) with $\phi = \phi_p$, as,

$$
\begin{align*}
  x &= o_p + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \cos \phi_1 \\
  y &= o_p + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \cos \phi_1 \\
  z &= h_p + \frac{\Delta \cos \phi_2}{\sin (\phi_2 - \phi_1)} \sin \phi_1
\end{align*}
$$

(6)

where $o_p = 1.342$ mm (0.053 in.) and $h_p = 6.740$ mm (0.265 in.).

Thus, for a given $\Delta$, the location of the object can be found by first measuring the radial position angle, $\theta'$, in the image plane. Since $\theta' = \theta$, and Eq. (5) can be used to convert the image distances $r'_{p1}$ and $r'_{p2}$ to $\phi_{p1}$ and $\phi_{p2}$, respectively, Eq. (6) can be applied.

The stereoscopic system can be calibrated to relate pixel locations in the annular images to field angles so that measurements can be made [17]. But distortions associated with mapping objects surrounding the lens to the image plane make it difficult to recognize and pinpoint shifted images of the same object without human intervention.

When the lens is translated, objects move radially and their aspect ratios remain constant. But the problem is that their sizes do not. This problem was solved for stereoscopic analysis by combining the methods of image linearization and digital correlation [18].

**CONCLUSION**

The PAL produces a panoramic image of its surroundings and a variety of methods can be applied for contouring. PAL systems can be applied to verify the condition of internal components or to predict problem areas and potential failure sites in structural, geomechanical, geotechnical and geoenvironmental systems. Commercial applications include pipeline inspection, detection of surface and near-surface cracks in weldments, detection of seams and foldovers in castings, monitoring of wear, detection of structural failures, and surface quality inspections.

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**REFERENCES**


