BOUNDARY CONDITION ANALYSIS OF A HARD X-RAY MIRROR

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ABSTRACT

This paper quantifies the residual Von Mises stresses and deformations in a hard X-ray mirror when the mirror is subjected to different constraints under gravity loading. A hybrid approach is employed which combines finite element analysis with experimental profile measurements taken from an earlier investigation conducted on a prototype formed by electroforming a thin shell on a cylindrical aluminum mandrel.

INTRODUCTION

The Constellation-X observatory (Con-X) [1] has been proposed to investigate black holes, Einstein's Theory of General Relativity, galaxy formation, the evolution of the Universe on the largest scales, the recycling of matter and energy, and the nature of "dark matter." It is a combination of several satellites orbiting in close proximity to each other and working in unison to generate the observing power of one giant telescope.

The telescopes aboard each satellite are designed to operate in the ~ 45 keV part of the electromagnetic spectrum and, as illustrated in Fig. 1, rely on four primary mirrors nested within each other. The mirrors follow mathematical curves - the parabola and hyperbola - derived by slicing through an imaginary cone at different angles.
X-ray telescopes are very different from optical telescopes, because with their high energies, X-ray photons will simply pass through a conventional mirror. To solve this problem, Con-X’s mirrors are cylindrically-shaped so that hard X-rays (~45 keV) are deflected into the instrument like stones skipping off water.

In a Wolter I design [2], like that proposed for Con-X, incoming photons undergo two reflections, the first from a parabolic surface and the second from a hyperbolic surface, to give an image that is essentially coma free. One approach developed to construct this design is to produce full-shell shallow-graze-angle gold-coated replicated mirrors by using electroformed nickel replication [3,4]. The advantage of electroforming is that complex coating procedures are avoided. The process lends itself readily to the multiple-mirror-module approach that small graze angles necessitate and the resulting shells provide excellent angular resolution that results in high sensitivity observations. This in turn translates directly into greater sensitivity through reduced focal spot size. Finally, with the use of high strength alloys one can achieve the stringent weight requirements of space-based missions.

During the mirror fabrication process, nickel mirror shells are electroformed onto a figured and superpolished aluminum mandrel from which they are later released by differential thermal contraction. Figure 2 illustrates that the resulting mirror shells are full circles of revolution.

Figure 1. The optical train of the Con-X space telescope.

Figure 2. A thin shell mounted in a support ring. Photo by Carl Benson, NASA/MSFC.
The axisymmetric geometry provides good structural stability permitting good figure accuracy, and hence very good angular resolution. Since nickel has a high density, researchers must make very thin shells to achieve the lightweight optics necessary to keep launch costs reasonable.

The shells must be strong enough to withstand the stresses of fabrication and subsequent handling without permanent deformation. They must also be electroformed in an ultra-low-stress environment to prevent stress-induced distortions once they are released. In short, the challenge is to maintain high angular resolution despite small weight-budget-driven mirror shell thickness. These requirements make shells extremely sensitive to the fabrication process and handling stresses.

The current mandrels used for electroforming represent conical approximations to Wolter-I geometry and typical metrology gives a performance prediction for the shells of around 8 to 10 arcsec half-power diameter (HPD), meaning half the reflected flux from a point source falls within this angular range. But X-ray tests reveal shell performances in the 13- to 15-arcsec range with modules running around 17-arcsec HPD [4,5].

In a recent study [6], laser techniques were used to profile an electroplated shell after it was manufactured. When the shell was positioned vertically on a flat surface, profile measurements showed that distortions in the mirrored surface created excessive optical distortions in the telescopic system. It was hypothesized that anomalies in the shape of the shell were due to residual stresses developed either during the electroplating process or while the shell was thermally removed from its mandrel.

A hybrid approach was used to study the problem and it was concluded that the unwanted deformation was due primarily to the residual stresses arising from the electrodisposition process as opposed to thermal extraction [7]. However, the model employed for this investigation constrained all six degrees of freedom at the support locations located along the rim of the mirror.

The current study is based on an analytical model that more accurately reflects the test set-up where reaction points are constrained only along the longitudinal and tangential directions. The difference in residual stress and residual deformation between these two boundary conditions are quantified.

**HYBRID ANALYSIS**

The hybrid approach consists of determining the Von Mises stress due to electroplating by subtracting the stress on the shell created by a gravity loading from the total stress computed based on profile measurements. Mathematically,

\[ \sigma_{\text{RES}} = \sigma_{\text{EXP}} - \sigma_{\text{GRAV}}. \]

The residual deformations are obtained in a similar manner by subtracting the deformation due to gravity from the deformation computed based on profile measurements. Again,

\[ \delta_{\text{RES}} = \delta_{\text{EXP}} - \delta_{\text{GRAV}}. \]

**CONICAL SHELL MANUFACTURING PROCESS**

Figure 3 shows the test article developed for this study. It consists of an ultra thin shell that is slightly conical in shape. The mirror is 23 in. long.

One half of the test article has a parabolic shape and an outer edge radius of 9.72 in. whereas the other half is hyperbolic with a radius of 9.707 in. Since the inner surface serves as a mirror for high resolution optical imaging, the shape of the shell must be carefully controlled during the manufacturing process.
The production of the mirror involves several steps. The first step is the fabrication of an aluminum mandrel with a radius of 0.004 in. below that required for the shell. The mandrel is then coated with 0.0039 in. of electroless nickel to give a hard surface suitable for polishing and the mandrel is then accurately figured using a cylindrical grinding machine. A mechanical super polish then takes place that is sufficient to ensure that scattering does not dominate the mirror’s performance up to the cut-off energy.

To prepare for electroforming, the surface of the mandrel is treated to form an oxide layer from which the shell can be easily released. Then the mandrel is immersed in the plating tank. A typical shell takes approximately 1 day to electroform, at which time the mandrel is taken from the bath, rinsed and dried and then cooled to separate the shell from the mandrel. This is done simply by immersing the assembly into a dewar of liquid nitrogen and then sliding the mandrel from the shell when release takes place.

A cross section of the five layers of interest and their thicknesses for the case considered are shown in Figure 4.

The upper layers of gold and cobalt-nickel are separated from the mandrel by injecting liquid nitrogen into its interior. The process utilizes the differences in the thermal coefficients of expansion of the materials and the relative bond strength between the layers to release the top two from the mandrel. The final configuration
consists of an ultra thin, open ended, conical shell, with a gold mirror on the inside and a cobalt-nickel substrate on the outside.

**EXPERIMENTAL MEASUREMENTS**

A series of experimental tests were conducted to support this research including thickness measurement, profiling, and yield stress determination. The shell was positioned vertically and profile measurements were taken on the outer surfaces along four meridians: 0, 90, 180, and 270 degrees. An average of the thickness was taken to obtain the variations in thickness over the height. Thickness values ranged from 0.0130 in. to 0.0134 in.

The shell profiles were measured in two separate runs [8]. The parabolic section was profiled first. Then the shell was inverted and the hyperbolic section profiled.

Figure 5 shows the deviation of each of the meridians from the design geometry in the parabolic section. The shell is supported at a height of zero (0.0 in.). If the shell contour were perfect, the curve would fall along the x-axis. The figure shows that the surface bulges outward at all the meridians.

![Figure 5. Parabolic shell profile.](image)

A maximum deformation equal to 352 µ inches occurs at 4.03 in. from the bottom in the 0 deg meridian. Three local maxima are observed on the curves and the deformation decreases to zero at a height of 11.5 in., which corresponds to the mid-section of the shell.

The deviation from the design geometry in the hyperbolic section is shown in Figure 6. Since the shell was inverted, the support corresponds to the height of zero (0.0 in.). Again, the shell bulges outward and a maximum deformation equal to 262 µ inches occurs at 0.62 in. from the bottom. Six local maxima are observed and the deformation decreases to zero at the mid-section, which corresponds to a height of 11.5 in.
A third test was performed to determine the yield and the ultimate stresses of the cobalt-nickel substrate following the ASTM Standard Coupon Specimen Test ASTM E8-99 procedure. The stress-strain curve for this test is plotted in Figure 7.

The test specimen failed at an axial load of 2,637 lb and the ultimate stress is estimated to be 225 ksi. The yield stress of the material was determined to be approximately 65 ksi based on the 0.2% offset method. As illustrated in Figure 8, test specimens did not exhibit any necking prior to failure and the fracture surface was normal to the longitudinal axis of the specimen, behavioral traits characteristic of a brittle material.
FINITE ELEMENT ANALYSIS

The finite element model (FEM) used to analyze the shell was generated using MSC/Nastran. The model is supported along the bottom edge at 12 equally spaced points. A gravity load was applied in the $-z$ direction.

Two boundary conditions for the models were used for comparison. The first being a full constraint condition, i.e., six degrees of freedom at the support points were constrained. The other boundary condition was a relaxed boundary condition where the tangential and longitudinal components were constrained and the radial and all rotation components were free. These models are depicted in Figures 9 and 10, respectively.

For the deformation loading, the worst case was selected for each section. The 0 deg meridian contour was used for the parabolic section and the 180 deg meridian contour for the hyperbolic section.
Since the shell is hollow, the model was meshed using 4-noded quadrilateral (CQUAD4) elements. A total of 18480 elements and 18720 nodes were generated. Material properties were obtained from the tensile test and assumed to be linearly elastic and isotropic.

DISCUSSION

The results (Patran plots) from the parabolic section for the residual stress for both the full constraint model and relaxed constraint model are shown in Figures 11 and 12 respectively. Residual displacement plots for this section are shown in Figures 13 and 14.
Figure 12. Residual stress/parabolic section/relaxed constraint.

Figure 13. Residual deformation/parabolic section/full constraint.

Figure 14. Residual deformation/parabolic section/relaxed constraint.
Similar plots were generated and studied for the hyperbolic section. The stresses and deformations for all of the load cases are summarized in Tables 1 and 2.

<table>
<thead>
<tr>
<th>Boundary Condition</th>
<th>Von Mises Stress</th>
<th>Deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Constraint</td>
<td>565 psi</td>
<td>3.76 x 10^{-4} in</td>
</tr>
<tr>
<td>Relaxed Constraint</td>
<td>772 psi</td>
<td>3.58 x 10^{-4} in</td>
</tr>
</tbody>
</table>

**Table 1.** FEM results comparison/parabolic section.

<table>
<thead>
<tr>
<th>Boundary Condition</th>
<th>Von Mises Stress</th>
<th>Deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Constraint</td>
<td>492 psi</td>
<td>2.20 x 10^{-4} in</td>
</tr>
<tr>
<td>Relaxed Constraint</td>
<td>662 psi</td>
<td>2.54 x 10^{-4} in</td>
</tr>
</tbody>
</table>

**Table 2.** FEM results comparison/hyperbolic section.

**CONCLUSION**

From the intermediate results, the difference between the gravity loading stresses in the parabolic section and the hyperbolic section was minimal as expected. Comparing the full constraint configuration to the relaxed constraint configuration in the parabolic section resulted in the Von Mises stresses increasing by approximately 37% and reducing the deformation by 4.8%. In the hyperbolic section, Von Mises stress increased by 34.5%. The deformation in the hyperbolic section increased by 15%.

More accurate modeling of the boundary conditions of the mirror has given better insight of the residual stresses developed in the structure. Changing from a fully constrained model to a less constrained configuration has changed the residual stresses significantly. The deformations were less affected by the change in boundary condition.

**ACKNOWLEDGEMENT**

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**REFERENCES**

[1] [https://conxproj.gsfc.nasa.gov/](https://conxproj.gsfc.nasa.gov/)


