

Vibration Analysis Using Digital Correlation

by

John A. Gilbert
University of Alabama in Huntsville
Huntsville, Alabama 35899

Donald R. Matthys
Marquette University
Milwaukee, Wisconsin 53233

T. Dixon Dudderar
AT&T Bell Laboratories
Murray Hill, New Jersey 07974

David L. Lehner
University of Alabama in Huntsville
Huntsville, Alabama 35899

Abstract

This paper demonstrates the use of a computer-based optical method for locating the positions of nodes and antinodes in vibrating members. Structured light patterns are projected at an angle onto the vibrating surface using a 35mm slide projector. The vibrating surface and the projected images are captured in a time averaged photograph which is subsequently digitized. The inherent fringe patterns are filtered to determine amplitudes of vibration, and computer programs are used to compare the time averaged images to images recorded prior to excitation to locate nodes and antinodes. Some of the influences of pattern regularity on digital correlation are demonstrated, and a speckle-based method for determining the mode shapes and the amplitudes of vibration with variable sensitivity is suggested.

Introduction

Analysis of vibrating surfaces by optical means is not new. In the late 1920's, for example, stroboscopic interferometry was used to study the surface of a vibrating quartz crystal[1,2]. An alternative interferometric method for analyzing the vibrations of piezoelectric crystals was subsequently reported[3]. In this technique, time averaged fringes were recorded over an exposure time equal to one or several periods of vibration. This led to improved methods for detecting relatively small amplitudes of vibration[4,5], including the holographic time-average technique[6]. Projected grating methods were later developed[7-9] to study the mode shapes and the relatively larger amplitudes of vibrating plates and other structural components. Many other techniques have since been reported.

For example, the authors recently suggested two alternative approaches in which structured light patterns were projected onto a vibrating surface using a 35mm slide projector[10]. In a technique called "line broadening," a single line was projected onto the surface at an oblique angle. Changes in effective line width were observed by viewing in a direction normal to the vibrating surface. These changes were calculated from comparisons between the initial line width recorded prior to excitation and the modulated line width recorded as the specimen vibrated. An alternative technique, called "shadow speckle metrology," utilized artificial speckles generated by splattering a clear glass slide with black paint. The patterns obtained by projecting the speckles onto a vibrating surface were captured either in a time averaged photograph which was subsequently digitized using a vidicon camera and slow digitizer, or in real time using the same digitizing system. The apparent in-plane movement of the projected speckles was computed by digitally correlating the stationary speckle pattern originally projected onto the sample with small subsets extracted from the digitized data recorded during vibration. Decorrelation took place in areas where motion occurred, and quantitative results showed this effect to be directly proportional to the amplitude of the displacement.

The present work is designed to quantify and utilize the results obtained in prior research on shadow speckle metrology, and to refine and demonstrate a full-field approach for characterizing mode shapes and measuring amplitudes of vibration.

Description of the Method

Figure 1 illustrates the geometrical configuration for the proposed approach in which a light source and camera are located at equal distances from the test surface. A structured light pattern is projected from point S onto the undeformed surface (AB) using a 35mm slide projector equipped with a glass slide containing the pattern. When the surface deforms or changes its location (to A'B'), the pattern appears to shift in the x-

direction. The corresponding displacement, u , is related to the out-of-plane displacement, w , by [11]

$$w = \frac{-u}{\tan \alpha + \tan \beta} \quad (1)$$

Although both α and β vary from point to point, the sum of their tangents is constant, and w is linearly proportional to u over the full field.

This same configuration can be used to study a steady state vibrating object by recording the surface with an exposure time long compared to the period of vibration. Following Sciammarella [8], assume the vibration to be sinusoidal of the form

$$A = A(x, y) \cos (\omega t + \psi) \quad (2)$$

where $A(x, y)$ is the amplitude function of the deflection of the surface, ω is the angular frequency and ψ an arbitrary phase constant.

The deflections of the surface with respect to the the coordinate system shown in Figure 1 can be expressed by

$$w(x, y, t) = z_0(x, y) + A(x, y) \cos (\omega t + \psi) \quad (3)$$

where z_0 is the stationary position of the object.

A grating of pitch p , with lines oriented parallel to the Y axis, can be projected onto the vibrating surface. In this case, the intensity distribution is given by

$$I(x, t) = I_0 + I_1 \cos \frac{2\pi}{p} [x - A(x, t) (\tan \alpha + \tan \beta)] + \dots \quad (4)$$

The time averaged exposure obtained by recording this intensity distribution over one complete period of vibration is given by,

$$E(x) = \left[I_0 + \sum I_n \left\{ \cos \frac{2n\pi}{p} [x - z_0 (\tan \alpha + \tan \beta)] J_0 \left[\frac{2n\pi}{p} A(x) (\tan \alpha + \tan \beta) \right] \right\} \right] t. \quad (5)$$

The above equation shows that the amplitudes of the harmonics forming the projected grating are modulated by a zero-order Bessel function of the argument γ_n . The argument depends on the local amplitude of vibration, and whenever the amplitude of vibration satisfies the condition

$$A(x) = \frac{p \bar{\gamma}_n}{2n\pi (\tan \alpha + \tan \beta)} \quad (6)$$

where $\bar{\gamma}_n$ are the roots of the zeroth-order Bessel function, i. e., $J_0(\bar{\gamma}_n) = 0$, the observed intensity corresponds to the average intensity, and the corresponding harmonic is eliminated.

Whenever the amplitude of vibration satisfies the condition

$$A(x) = \frac{p \bar{\gamma}_n}{2n\pi (\tan \alpha + \tan \beta)} \quad (7)$$

where \bar{Y}_n are the maxima of the zeroth-order Bessel function, the corresponding harmonics will be present. If an observer looks at the surface, he will see areas of average intensity (dark fringes) and areas where the grating is reproduced (light fringes).

The fringe patterns corresponding to the different harmonics of the grating can be observed by filtering. The intensity corresponding to the nth order is,

$$I(x) = k^2 J_0^2 \frac{2n\pi}{p} A(x) (\tan \alpha + \tan \beta) \quad (8)$$

Therefore, in the simple case where the structured light pattern consists of a line grating having pitch p oriented parallel to the Y axis, the filtering process produces moire fringes whose intensity is modulated by the square of the zero-order Bessel function.

The aforementioned arguments can be extended to include the case in which a quasi-random pattern of splattered paint (speckles) is projected onto the surface by considering the speckle pattern to be composed of a finite number of gratings having differing pitch and orientation. Moire fringes and the modulation in the intensity of the time averaged pattern become less pronounced but they are still present and can be significantly enhanced by filtering. The sensitivity of measurement is governed by the position of the aperture used to sample the diffraction halo in the transform plane[12].

In addition to filtering, computer programs can be used to extract subsets surrounding selected points in an initial recording of the stationary test surface and the projected pattern. When these subsets are compared numerically with subsets extracted from the time averaged image, correlation values lying between -1.0 and +1.0 can be computed. Since higher values of correlation coefficients indicate better agreement, this information can help to define the mode shape of the vibrating component. The following series of tests demonstrates that digital correlation can be used to locate nodal positions and mode shapes of vibrating members.

Experimental

The set-up shown in Figure 2 was designed and built to measure the mode shapes and vibration amplitudes of a vibrating structural component. A slide containing a structured pattern is projected from the right at 45 degrees with respect to the normal to the test surface ($\alpha = -45^\circ$). Images are recorded with a 35mm camera positioned normal to the surface of the test specimen.

The test specimen consisted of a tensioned pliable rubber strip 50mm (1.97 inches) wide, 850 mm (33.46 inches) long, and 0.81 mm (0.032 inches) thick, whose surface was painted flat white to increase contrast of the projected pattern. The strip was held fixed at both ends. Its center was attached to an audio speaker which was driven through a variable amplifier by a signal generator capable of producing a sinusoidal wave. This arrangement was used to excite different transverse modes of vibration. The motion of points along lines parallel to the longitudinal axis of the strip exhibit behavior analogous to the motion of points along a vibrating string as described in the classic Melde vibration problem.

Three different patterns (parallel lines, regular dots, and quasi-random speckles) were projected onto the surface of the strip. Photographs were taken of the stationary strip using Kodak TMAX 100 film with an exposure time of 1/2 second. The specimen was excited at a frequency of 36 Hertz, and time averaged patterns for each projected pattern were recorded over the 1/2 second interval.

All tests were performed with the specimen excited at an antinodal location; five periods were observed over the length of the specimen. The photographs covered a portion of the specimen containing two nodes and one antinode with a peak to peak amplitude of approximately 4 mm. The 35mm negatives of these images were projected onto a screen, digitized and stored in a PDP 11/73 computer as 256 x 256 pixel arrays with 256 grey levels.

An initial test was conducted using a grid consisting of twenty equispaced parallel lines projected vertically across the surface of the strip ($p = 2.5$ mm). This test was designed to illustrate that filtering can be used to extract time averaged moire patterns, and to

quantify the effect caused by pattern interaction on digital correlation techniques. Figure 3 shows the time average recording containing a moire pattern with fringes of varying contrast. These fringes are characterized by Equation (8). The illuminated area to the right of the specimen contains test patterns and a scale for calibration of the optical system, and can be used to study the size of the projected pattern relative to the pixel size; the vertical span of the image covered 256 rows. Fringe contrast was enhanced by filtering the image. Figure 4 shows the filtered pattern obtained by isolating the +1 order in the transform plane.

Computer programs were written to digitally correlate the initial pattern recorded in the static condition with the modulated patterns recorded while the object was vibrating. Software was written to search for the maximum correlation coefficient in regions surrounding points located along the vertical center line of the specimen (parallel to the longitudinal axis of the strip). The purpose of this search was to take into account rigid body motion of the specimen as it went into vibration, and any misalignment that might arise from inaccurate registration of the time-average photographs during digitization. The results of this analysis are shown in Figure 5; the influence of the moire effect on the correlation values is clearly visible.

In the second test, a regular pattern of equispaced dots (with spacing equal to that of the lines used in the first test) was projected onto the surface. Images were recorded for the static and dynamic cases. The moire effect is much less pronounced in the time averaged recording shown in Figure 6, but can be enhanced by filtering the image. The corresponding plot of correlation coefficients versus position is shown in Figure 7. A comparison with Figure 5 shows that correlation coefficients are lower at locations corresponding to the moire fringes, and that this effect is more pronounced for higher order fringes.

A final test involved projecting a quasi-random pattern of speckles onto the test surface. Figure 8 shows the time-averaged photograph recorded over 1/2 second when the strip was vibrating with peak to peak amplitude of 4 mm at 36 Hertz.

The speckle pattern is equivalent to a finite number of gratings of different orientation and pitch. Moire fringes caused by this superposition merge and no distinct fringes result. However, the fringe pattern contained in Figure 5 is inherent in the specklegram, provided that the same spatial frequency ($1/p$) of the lines in the grating is contained within the range of speckle sizes. This information can be extracted by sampling the diffraction halo using an aperture positioned at the same location in the transform plane. The result of performing this operation on Figure 8 is shown in Figure 9. Contrast is poor when this result is compared to the filtered pattern shown in Figure 4, mainly because only a few speckles in the pattern diffract light through the filtering aperture.

A plot of correlation coefficients for the case in which speckles are projected is shown in Figure 10. In this case, correlation coefficients maximize at the nodes and tend to fall off toward the antinodes. The local peaks observed in Figures 5 and 7 begin to disappear.

Discussion

In Figure 3, the intensity is modulated by the square of the zero-order Bessel function, and the brightest of the fringes in the pattern appear at the nodes; fringe contrast falls off toward the antinodes. The analysis of this pattern is relatively straightforward, and both the amplitude and the corresponding mode shape can be determined. However, these determinations may prove difficult, especially in a more complicated situation where the object brightness may vary and/or no nodes exist in the region studied. In these cases, a combination of filtering and numerical correlation may aid in determining vibration amplitudes and the corresponding modal shapes. The results obtained in this study indicate that this approach can be optimized using a projected speckle pattern containing individual speckles with random size and spatial distributions. The more random the pattern, the more accurately the plot of correlation coefficient versus position will reflect the mode shape. However, limitations on the randomness are imposed, since large speckles may obscure a portion of the region of interest, and no information can be obtained for speckles which fall below the resolution limit of the recording system. These limitations determine the range over which measurements can be made.

Conclusion

This paper demonstrates that digital correlation can be used to determine the mode shapes of a vibrating structural component. Even though the method was applied to a simple vibrating strip, the results of the study form the basis for the development of an automated detection system which could be portable, compact, fast, and easy to operate. The authors are currently refining algorithms for digitally filtering time averaged patterns.

Admittedly, a technique based on numerical correlation and digital filtering will be computer intensive but it can be applied to any surface, it is full-field, non-contacting and non-destructive, the analysis can be completely automated, and tests may be conducted under normal room lighting. In addition, sensitivity may be altered by varying the directions of observation and projection or by adjusting the position of the aperture in the transform plane.

Acknowledgements

The authors would like to thank MSFC/NASA and the Alabama Research Institute for supporting portions of this work.

References

1. Dye, D.W., National Physical Laboratory Report, 1928.
2. Dye, D.W., "The modes of vibration of quartz piezo-electric plates as revealed by an interferometer," Proc. Royal Soc., London 138A, 1932.
3. Osterberg, H. "An interferometer method of studying the vibrations of an oscillating quartz plate," J. of the Opt. Soc. of Am., 22(19): 19-35, 1932.
4. Thornton, B.S., Kelly, C.J., "Multiple-beam interferometric method of studying small vibrations," J. of the Opt. Soc. of Am., 46(3): 191-194, 1956.
5. Goldberg, J.L., "New method for the accurate standardization of mechanical oscillations of small amplitude," The J. of the Acoustical Soc. of Am., 36 (10): 1922-1925, 1964.
6. Powell, R.L., Stetson, K.A., "Interferometric vibration analysis by wave front reconstruction," J. of the Opt. Soc. of Am., 55(12): 1593-1598, 1965.
7. Hazell, C.R., Niven, R.D., "Visualization of nodes and antinodes in vibrating plates," Experimental Mechanics, 8(5): 225-281, 1968.
8. Sciammarella, C.A., "Analysis of the vibration of plates by means of the moire method," Metodi ed applicazioni dell'analisi sperimentale delle tensioni, Palermo, 1972, pp. 15-21.
9. Hung, Y.Y., Liang, C.Y., Hovanesian, J.D., Durelli, A.J., "Time-averaged shadow-moire method for studying vibrations," Applied Optics, 16(6): 1717-1719, 1977.
10. Lehner, D.L., Dudderar, T.D., Matthys, D.R., Gilbert, J.A., "Vibration analysis using shadow speckle metrology and line broadening techniques," Proc. of the 1987 Spring Conference on Experimental Mechanics, Society for Experimental Mechanics, Houston, Texas, June 15-19, 1987, pp. 55-61.
11. Gilbert, J.A., Matthys, D.R., Taher, M.A., Petersen, M.E., "Profiling structural members using shadow speckle metrology," Optics and Lasers in Engineering, 8(1): 17-27, 1988.
12. Cloud, G.L., "Practical speckle interferometry for measuring in-plane deformations," Applied Optics, 14(4): 878-884, 1975.

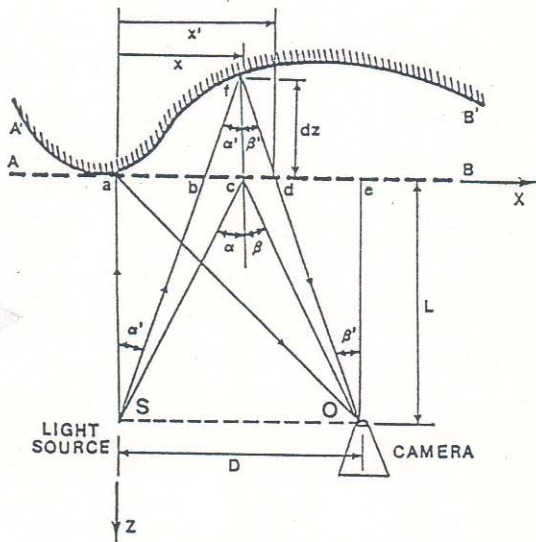


Fig. 1 Schematic for deflection analysis using projected patterns.

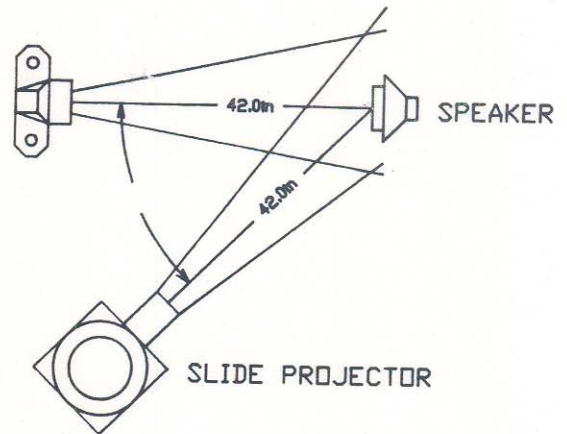


Fig. 2 Experimental set-up for vibration analysis.

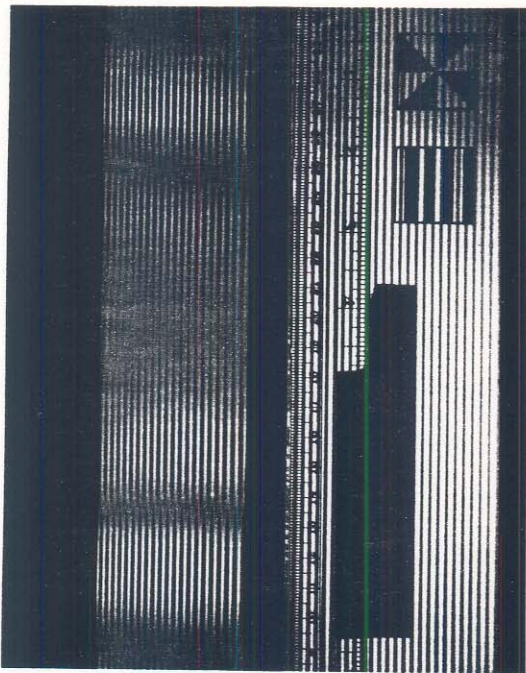


Fig. 3 Time averaged recording of the vibrating strip and projected lines.

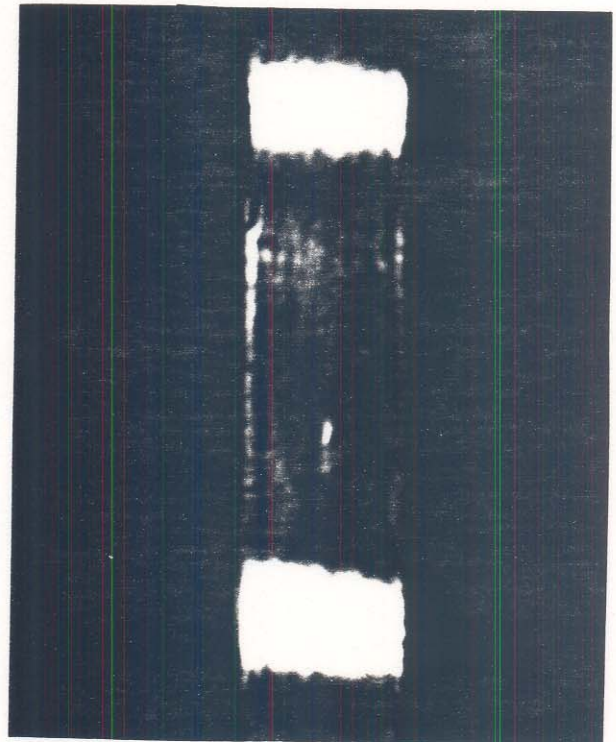


Fig. 4 Filtered image of the time averaged pattern shown in Figure 3.

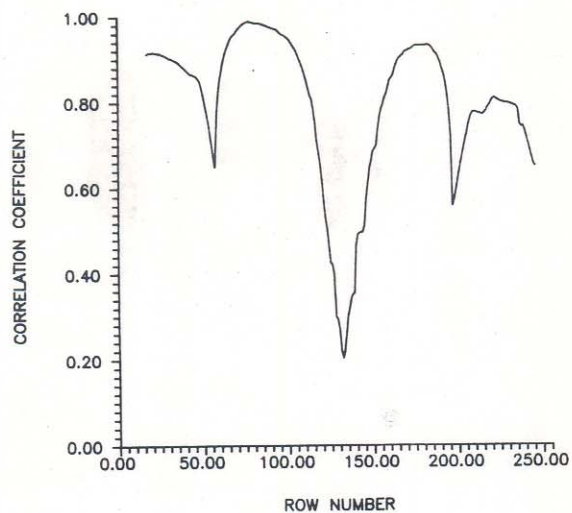


Fig. 5 Correlation coefficients versus position along the vertical center line of the vibrating strip for projected lines.

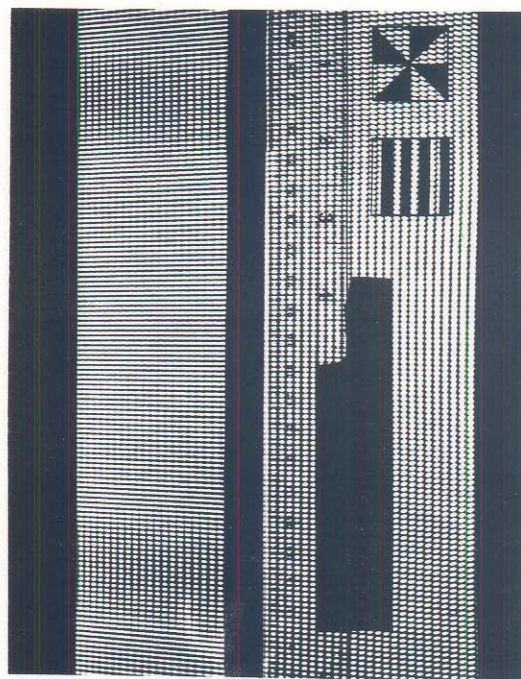


Fig. 6 Time averaged recording of the vibrating strip and projected dots.

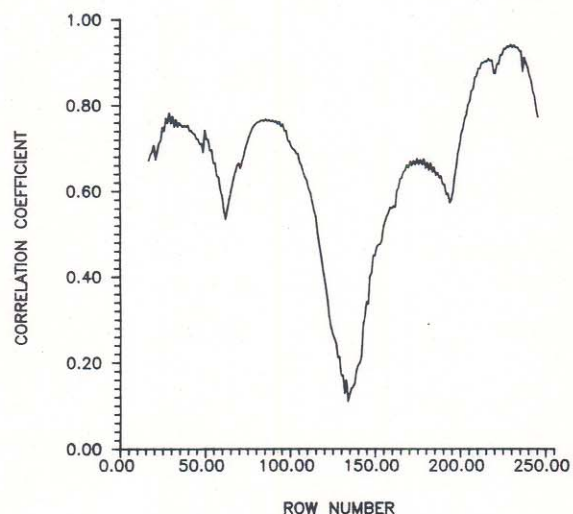


Fig. 7 Correlation coefficients versus position along the vertical center line of the vibrating strip for projected dots.

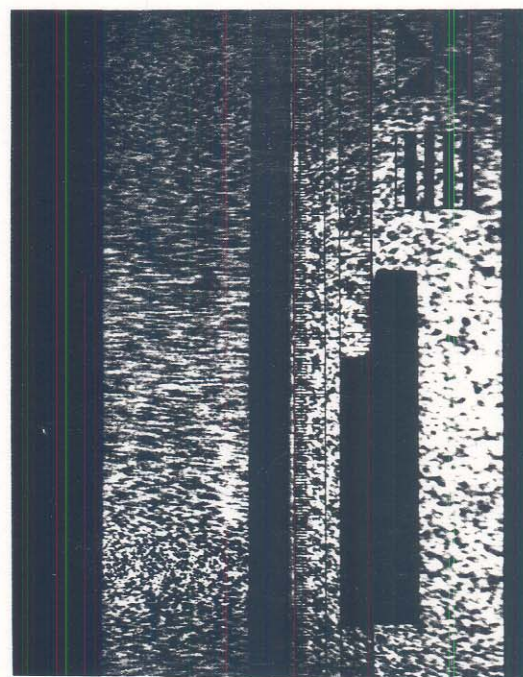


Fig. 8 Time averaged recording of the vibrating strip and projected speckles.

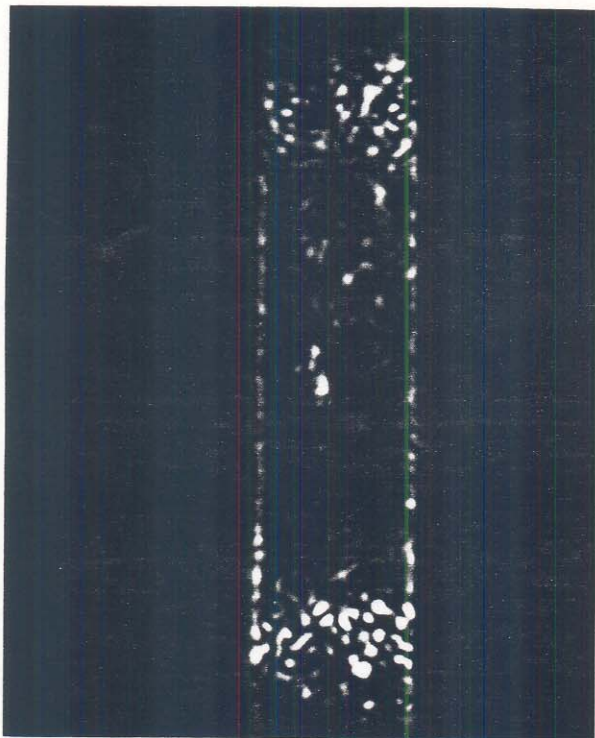


Fig. 9 Filtered image of the time averaged pattern shown in Figure 8. The image was obtained with the same bandpass filter used to obtain Figure 4.

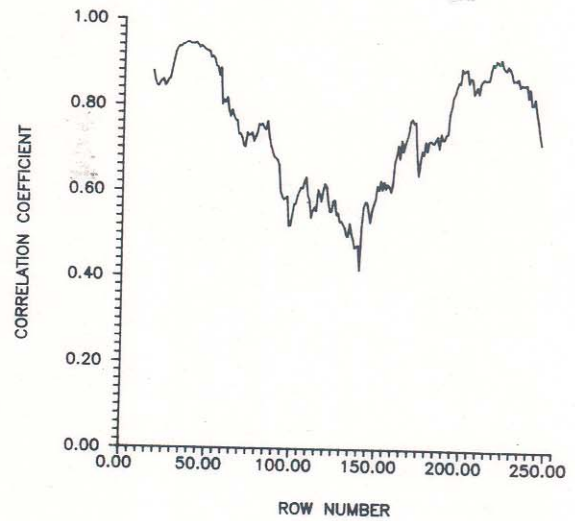


Fig. 10 Correlation coefficients versus position along the vertical center line of the vibrating strip for projected speckles.