Measurement of Two-Dimensional Fluid Flow by Digital Correlation of Scattered Light

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Abstract

Digital correlation is a pattern matching procedure. This procedure has recently been used in two-dimensional fluid flow measurements. Two images of a seeded fluid are taken and the intensity distribution of a small window in the first image is matched in the second image using a statistical correlation method. A series of structure functions have been implemented to improve the accuracy and reliability of the correlation method and new algorithms have been developed to reduce the time required for the correlation computations.

Introduction

Full-field two-dimensional fluid flow velocity measurement plays an important role in the understanding of time dependent flows such as the formation and breakdown of vortices, dynamic stall, and turbulence. In recent years, with the advances in microcomputers and in laser technology, new methods have been developed to determine full-field two-dimensional fluid flow velocity. These methods are generally called full-field laser velocimetry. In most cases, a transparent fluid is seeded with particles or tracers which follow the flow. A full-field fluid flow velocity distribution map can be obtained by tracing these seed particles in the fluid. The movement of these particles is observed in a reference plane which is usually illuminated using a thin sheet of laser light passing through the fluid volume parallel to the motion.

Traditional procedures for analyzing laser specklegrams are mainly divided into two approaches: methods that use pointwise interrogation to obtain Young's fringes and methods that utilize fringes resulting from optical filtering in the Fourier plane. Although significant progress has been made in improving and automating these approaches, each has its own limitations.

The Young's fringes technique can give the direction and the magnitude of flow velocity, but with an ambiguity in the sign of flow direction. To decide the sign of flow direction, special means[1,2,3] must be used. Another major disadvantage of the Young's fringes method is that the technique can not measure displacements smaller than the diameter of the images obtained from the microparticles seeded in the fluid, and if the particles are made very small, the intensity of the scattered light becomes weak. In addition, the use of laser light to interrogate the specklegram containing the image data introduces secondary speckle into the Young's fringes being measured, and limits the accuracy with which their position can be determined.

The method of filtering in the Fourier plane, on the other hand, gives a full-field measurement immediately, but only for a particular component of the velocity vector. To obtain more information about the fluid flow, a velocity-selecting aperture must be moved radially and tangentially around the center of the diffraction halo produced by the speckle pattern being studied. The need for laser illumination will again introduce secondary speckle which, if the aperture is kept small to select a narrow frequency range, makes the fringes in the filtered image difficult to read.

Recently, a method that employs digital correlation and a combination of aperiodic images has been developed.[4] A series of pairs of pictures of the fluid are taken with aperiodic time intervals between each pair, while the time intervals within each pair of pictures are kept the same. The pictures are separated into two groups with the first image in each pair forming one group and the second image in each pair forming the other group. Each group of images is combined into one single image by arithmetic adding of the image intensities from each picture so that two combined images are produced. The digital correlation technique is used to analyze the speckle displacement between the two combined images. A number of windows are opened in the first image and moved over the second image to find their best match so that the displacements of each window can be measured.[5] Since the time interval between each pair of images is known, a full-field fluid flow pattern can be obtained. The formula to determine the correlation coefficient, ρ, for a data window taken from the first image and centered over location (m,n) in the second image is[6]

$$\rho(m,n) = \frac{\sum_x \sum_Y [f(x,y) - \langle f \rangle][w(x-m,y-n) - \langle w \rangle]}{\{\sum_x \sum_y [f(x,y) - \langle f \rangle]^2 \sum_x \sum_y [w(x-m,y-n) - \langle w \rangle]^2\}^{1/2}} \tag{1}$$

where $w(x,y)$ are the intensity values of the window opened in the initial image, $f(x,y)$ are the intensity values of the second image for those locations under the window values $w(x,y)$, $\langle w \rangle$ is the average intensity values of the window, and $\langle f \rangle$ is the average intensity value of the region located under the window. The maximum correlation value in the search region indicates the best match of the chosen subset (window) from the initial image as located in the second image.

Unfortunately, the straightforward application of this correlation method by itself may not always give accurate results. For example, Figure 1 is a velocity distribution map for a counter-clockwise water flow obtained by using Equation (1). Each arrow in the map represents the fluid velocity at the midpoint of the arrow. It is quite obvious that some of the velocity arrows are wrong. This paper discusses the cause of these errors and proposes several "structure functions" to make the correlation method work in fluid flow measurement. In addition, this paper introduces some algorithms to shorten the calculation of the correlation values so that the speed of the data analysis can be improved.

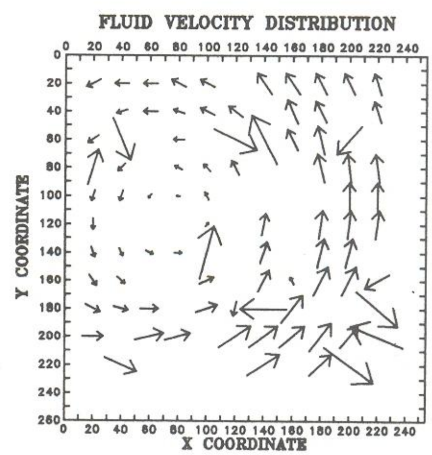

Figure 1. A map of two-dimensional fluid flow velocity distribution. No structure functions have been used.

Structure Functions

Since the digital correlation method depends on tracing the motion of the microparticles in the fluid to determine the fluid velocity, any window selected in the first image should contain some particle images. In Figure 1 a window centered at (120, 60) gives a velocity arrow that is obviously wrong. A close look at this window's intensity distribution is shown in Figure 2, which reveals that the window is almost empty. Most of the pixels inside the window have very low intensity values which can be regarded as noise. There are very bright pixels only at the far right corner. Lack of content in a window is one of the reasons which cause spurious results. If a window has no speckles, it is meaningless to find its match in the second image because it does not have any information about the particle movement. Furthermore, if a window has only one speckle, not enough to make the window unique, correlation methods are likely to find very good matches at several locations in the second image. This will also lead to incorrect conclusions. Figure 3 is a contour map of correlation values over the search region for the window shown in Figure 2. The correlation values are plotted as contours, with the outer contour having a value of 0.45 and each succeeding interior ring representing an increment in correlation value ρ of 0.05. It is obvious that there are several positions with high correlation values in the map. To obviate this problem, two thresholds are used to determine whether a window has enough content. The intensity threshold defines the minimum value required for a pixel to be regarded as a bright pixel indicating a microparticle. The number threshold defines the minimum number of bright pixels required for a window to be classified as having enough content. A satisfactory window should meet both requirements. To implement these thresholds, a computer program is used to carry out the analysis. The program counts the number of pixels within the window that have an

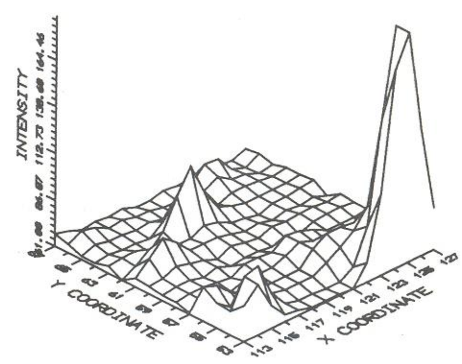

Figure 2. Intensity distribution for a window centered at location (120,60).

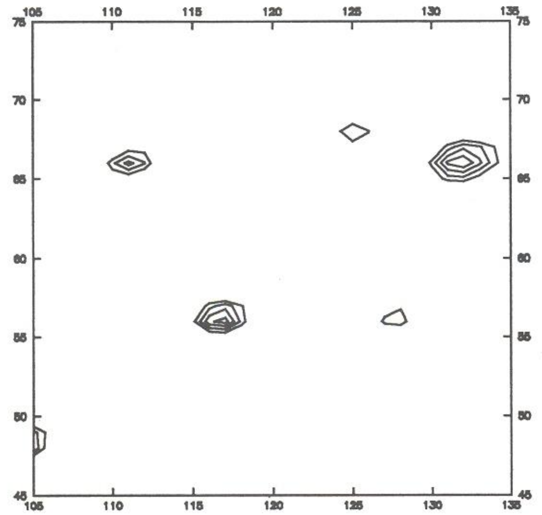

Figure 3. Correlation values over the search region for window shown in Figure 2.

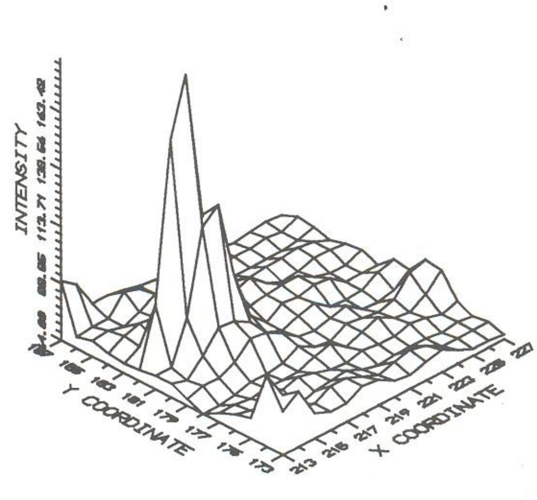

Figure 4. Intensity distribution map of the window centered at (220,180). There is only one peak.

intensity value higher than the intensity threshold. If the number is less than the specified number threshold, the window will be regarded as unsuitable and will not be included in the correlation evaluation.

A good window should not only have enough content but also should have enough structure to distinguish itself from other areas in the initial image so that only one position in the second image will be found to have a high correlation value. In many cases, when an unreasonable result appears, the window contains less than two speckles. A window containing no speckles has no information about the fluid flow, while a window with only one speckle will not produce a unique result because this speckle will produce a high correlation value whenever it matches a speckle with a similar size in the second image. Therefore, to define a window as having enough structure, it should have at least two speckles, or in other words, the window's intensity distribution map should contain at least two peaks with each peak representing a speckle. The more speckles in a window, the more structure it has. Figure 4 is an intensity distribution map of the window chosen to obtain the (erroneous) velocity arrow located at (220, 180) in Figure 1. There is only one peak in the window. Figure 5 shows the correlation values over a search region. Because the window does not have enough structure, there are several places with a high correlation value in the search region. Figure 6 shows the intensity distribution of a window centered at (120,200) which has two peaks. The corresponding

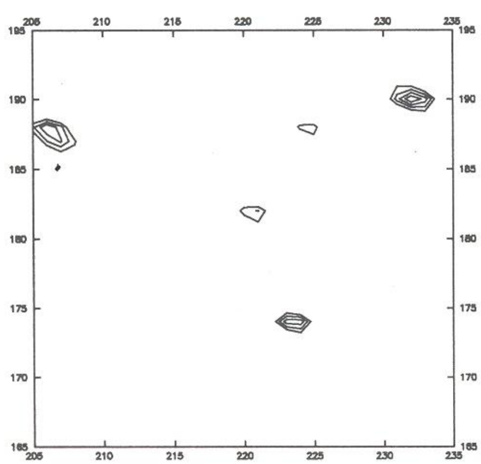

Figure 5. Correlation values over the search region for the window shown in Figure 4.

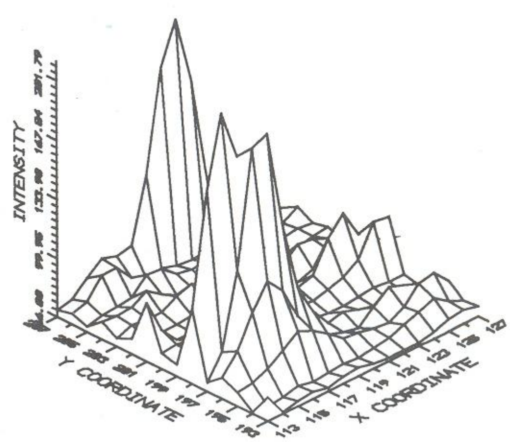

Figure 6. Intensity distribution of a window with two peaks.

map of correlation values presented in Figure 7 shows that there is only one position with a high correlation value. To implement the structure function described earlier so as to keep the second window and reject the first, a computer program first counts for each window the number of pixels with an intensity value higher than a chosen threshold. Then the program finds the highest peak in the window and from this peak, it searches the neighborhood of the peak for the boundary where the pixel intensities begin to fall below the intensity threshold. Once the boundary is found, the computer counts the number of pixels inside the boundary and compares this number with the total number of pixels above the threshold in the window. If there are a comparable number of bright pixels located outside the boundary, it means that there is at least one more peak in the window. If it is found that most of the bright pixels are located within the boundary and there are not enough left to be caused by a second particle, it can be concluded that the window does not have enough structure, and should not be included in the correlation evaluation.

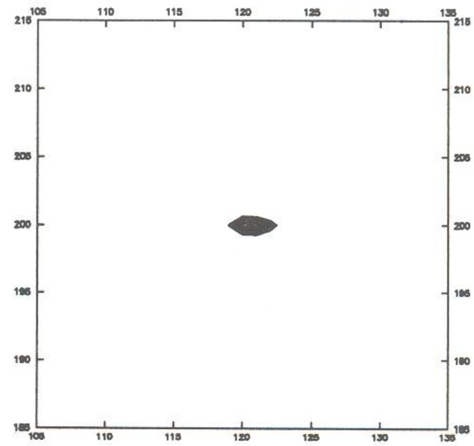

Figure 7. Correlation values over the search region for the window shown in Figure 6.

Some of the images obtained have a considerable amount of fluctuation in intensity due to background or stray light. This fluctuation, or noise, degrades the quality of the images and impacts strongly on the accuracy of the measurement. This noise often appears as image particles to the correlation algorithm and can spoil the measurement by giving spurious results. A filter to reduce the noise should be developed and applied to the image. This filter can be a simple clipping threshold for the intensity values in an image. When an image is being processed, the computer program compares each pixel value with this clipping threshold. If the pixel value is smaller than the threshold, the computer will regard this pixel as noise and set the pixel value to zero which means completely dark. If the pixel value is equal to or greater than the threshold, it will be treated as a part of a particle image and kept unchanged. In most cases, the noise intensity level is very close to that of the background, and therefore a reasonable approach is to calculate the average pixel value in an image and then set this value as the threshold. Because the real particle intensity values are usually much higher than those of the background and noise, the average intensity value of an entire image will be between the particle intensity values and noise intensity values so that most noise can be filtered out. Figure 8 is an image which has been processed by the noise filter. The right corner of the image is still blurred by noise because noise here is as strong as the speckles. A window is selected in the lower right corner of the image and its intensity distribution is shown in Figure 9. Although the window contains enough content and structure, it is heavily contaminated by noise. The characteristic which distinguishes a blurred window from a good one is that the blurred window has a large number of bright pixels. A computer program is used to count the number of the

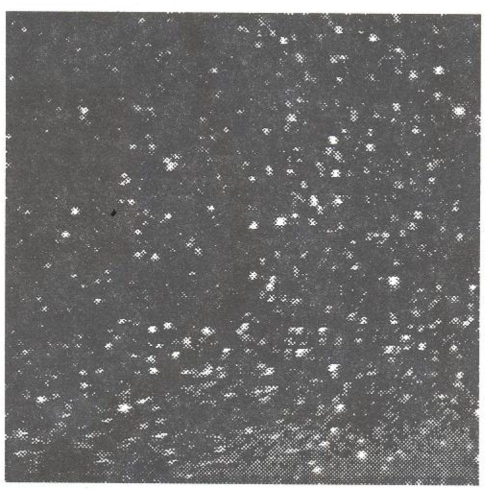

Figure 8. A multiple exposure image. The bottom edge of the image is blurred by noise.

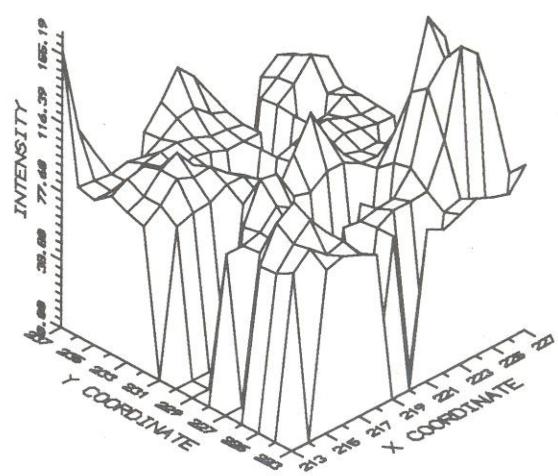

Figure 9. Intensity distribution of a window chosen from the lower right corner of Figure 8. The window is heavily blurred so there is almost no black background.

bright pixels in a window. If there are too many bright pixels in a window, the computer program will regard it as contaminated and discard it.

In full-field fluid flow measurement a number of windows are chosen at locations distributed over the entire initial image to produce a complete picture of the fluid velocity distribution. For each window, the structure functions mentioned above are applied to verify its quality. Because the poor quality windows will be discarded by the structure functions, only a subset of the original windows will be used in the digital correlation evaluation. The consequence of simply dropping windows is that the full-field measurement will have gaps and become incomplete. To ensure a full-field fluid velocity measurement with few gaps, a window searching program is developed to search for good windows. If a window does not meet the requirements of the structure functions, it will be discarded and a neighboring position will be chosen as the new window position and the entire testing process will be repeated. Searching will continue until a satisfactory window is found or every window position has been tried in a predefined region surrounding the original rejected window.

Fast Correlation Algorithm

Another major problem with the digital correlation method is that it is very time consuming, since a large amount of calculation is needed to evaluate the correlation coefficient ρ. For example, to obtain 121 correlation values using windows of 15 by 15 pixels over a search region of 31 by 31 pixels, takes 20 to 30 minutes on a 386 microcomputer. By applying two additional algorithms, this time has been reduced to about 5 minutes.

Figure 10 represents the distribution of correlation values over a region in the format of a contour map. The maximum correlation value occurs at the position where the best match is found. Since the lowest correlation values shown are 0.4, most of the region is below that cutoff. For an area with low correlation values, it is not necessary to evaluate the correlation coefficient at every position, and the correlation calculation may be performed at more widely spaced intervals. This assumes that the correlation values will not change abruptly between neighboring positions. With this assumption, if the correlation value ρ at a certain position is very low the neighboring positions can be skipped because the ρ value at those positions will not be much higher. By doing so, the search range can be effectively cut to approximately half of its original size in each search direction, and so only about one fourth of the calculations need be done. This produces a dramatic improvement in correlation speed.

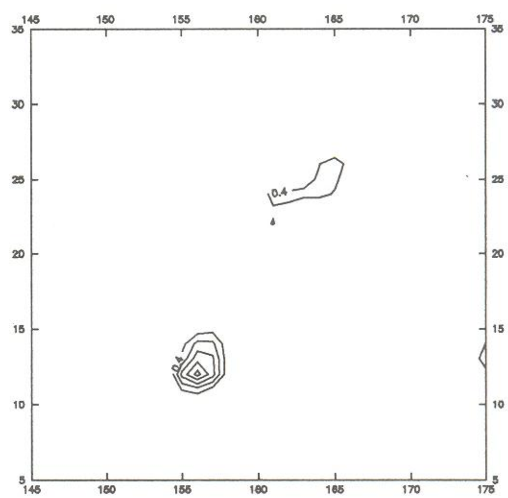

Figure 10. The contour map of correlation distribution over the search region.

Another way to improve the speed is to use a modified formula in the correlation calculation. The standard equation for digital correlation evaluation is Equation (1). A computer program is used to carry out the calculation based on Equation (1) but with some modifications. When evaluating the correlation coefficient ρ at a certain position in the second image, the computer first reads the pixel values of the window into an array **U**. If the size of the window is W by W, then the array **U** has W^2 elements. Each element contains a pixel value in the window. The pixels from the region which is under the window in the second image are read into an array **V**. The array **V** is of the same size as array **U**. Now let u_i define the *ith* element of array **U**, v_i be the *ith* element in array **V**, $<u>$ the average value of array **U**, and $<v>$ the average value of array **V**. Then Equation (1) can be expressed as:

$$\rho = \frac{\sum_{i=1}^{W^2}(u_i - <u>)(v_i - <v>)}{\left[\sum_{i=1}^{W^2}(u_i - <u>)^2 \sum_{i=1}^{W^2}(v_i - <v>)^2\right]^{1/2}} = \frac{\sum_{i=1}^{W^2}(u_i - <u>)v_i - <v>\sum_{i=1}^{W^2}(u_i - <u>)}{\left[\sum_{i=1}^{W^2}(u_i - <u>)^2 \sum_{i=1}^{W^2}(v_i^2 - 2v_i<v> + <v>^2)\right]^{1/2}} \quad . \tag{2}$$

Since

$$\sum_{i=1}^{W^2}(u_i - <u>) = \sum_{i=1}^{W^2} u_i - \sum_{i=1}^{W^2} <u> = W^2 <u> - W^2 <u> = 0 \tag{3}$$

then

$$\rho = \frac{\sum_{i=1}^{W^2}(u_i - <u>)v_i}{\left[\sum_{i=1}^{W^2}(u_i - <u>)^2 \left\{\sum_{i=1}^{W^2} v_i^2 - <v>^2 W^2\right\}\right]^{1/2}} . \tag{4}$$

Defining

$$u_i' = u_i - <u>; \qquad a = \sum_{i=1}^{W^2}(u_i - <u>)^2 = \sum_{i=1}^{W^2}(u_i')^2; \qquad b = \sum_{i=1}^{W^2} v_i^2 \tag{5}$$

the correlation coefficient ρ can then be expressed as:

$$\rho = \frac{\sum_{i=1}^{W^2} u_i' v_i}{\left[a\{b - <v>^2 W^2\}\right]^{1/2}} . \tag{6}$$

The difference between Equation (1) and Equation (5) is that in Equation (1), for every position (m,n), $(y_i - <y>)$ and $(y_i - <y>)^2$ should be calculated while in Equation (5) the corresponding calculations can be omitted. This saves a lot of work in the correlation evaluation. Furthermore, it is not necessary to start fresh each time in calculating the square sum of y_i because there is a lot of redundant calculation between neighboring positions. Assume that $f(x,y)$ is the pixel intensity in the second image. The center of a window is now at the location (m,n). The size of the window is W by W (W is an odd number). Then the sum of the squares of v_i in Equation (5) is:

$$\sum_{i=1}^{W^2} v_i^2 = \sum_{x=m-\frac{W-1}{2}}^{m+\frac{W-1}{2}} \sum_{y=n-\frac{W-1}{2}}^{n+\frac{W-1}{2}} f^2(x,y) . \tag{7}$$

When the window moves to the next horizontal position $(m+1,n)$, the sum of the squares of v_i is:

$$\begin{aligned} &\sum_{x=m+1-\frac{W-1}{2}}^{m+1+\frac{W-1}{2}} \sum_{y=n-\frac{W-1}{2}}^{n+\frac{W-1}{2}} f^2(x,y) \\ &= \sum_{x=m-\frac{W-1}{2}}^{m+\frac{W-1}{2}} \sum_{y=n-\frac{W-1}{2}}^{n+\frac{W-1}{2}} f^2(x,y) - \sum_{y=n-\frac{W-1}{2}}^{n+\frac{W-1}{2}} f^2\left(m - \frac{W-1}{2}, y\right) + \sum_{y=n-\frac{W-1}{2}}^{n+\frac{W-1}{2}} f^2\left(m + 1 + \frac{W-1}{2}, y\right) . \end{aligned} \tag{8}$$

So when the window moves to the next horizontal position, only the sum of the squares of the left column and the right column need be recalculated. This is only a small fraction of the calculations otherwise required. Similarly, if the window moves to the next vertical position, only the top row and the bottom row need to be recalculated.

In analyzing the data to determine the fluid flow, both algorithms were applied, which reduced the time required by about a factor of four, compared to omitting the algorithms.

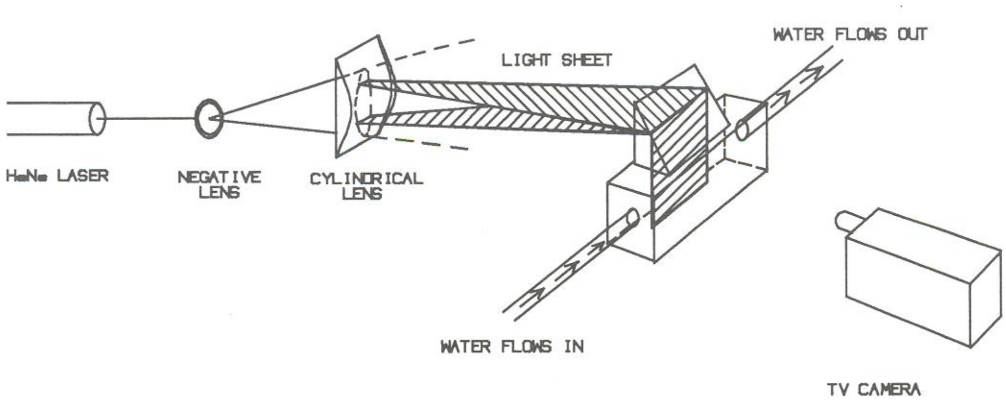

Figure 11. Experimental setup for fluid flow experiment.

Experiment

To produce a flow field for the experiment, tap water was pumped through a rectangular plastic cell. The flow could be controlled by varying the pump speed and the locations of the feed lines to the cell. The water was seeded with microparticles so that when illuminated they produced scattered light to form the particle images. The light source for illumination was a 10 mW He-Ne laser. The laser beam passed through a cylindrical lens which focused the beam into a horizontal sheet of light. A prism placed on top of the cell reflected the light straight down to illuminate a vertical plane in the test cell. A CCD camera controlled by an image processor acquired the speckle patterns. The area of fluid studied in this experiment varied between 0.5 cm × 0.5 cm and 5 cm × 5 cm, depending on the size of the seed particles, which ranged from 15.7 microns to 99 microns. The experimental setup is shown in Figure 11.

Results

The digital correlation method can be used to measure both one-dimensional and two-dimensional fluid flow. Figures 12 and 13 are two velocity distribution maps of water flow. Each arrow in the maps represents the fluid velocity at the midpoint of the arrow. Figure 12 is a one-dimensional water flow. The transverse area of fluid under study is 4.0 cm × 3.5 cm. The two-dimensional water flow, shown in Figure 13, flows in a counter-clockwise loop.

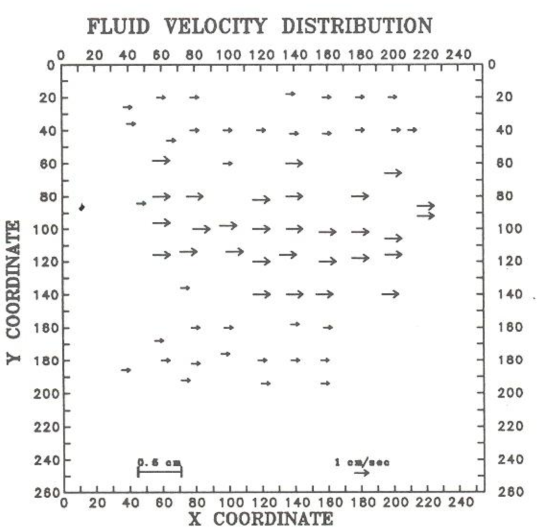

Figure 12. One-dimensional fluid flow velocity distribution.

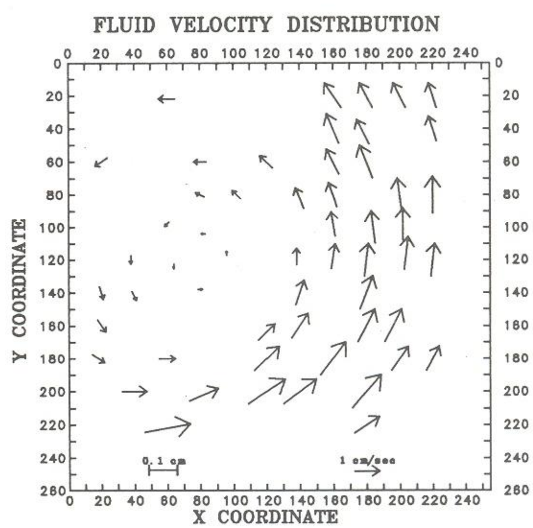

Figure 13. Two-dimensional fluid flow velocity distribution.

Conclusions

The digital correlation method has been shown to be a very effective means for obtaining full-field two-dimensional fluid flow measurements. The experimental setup is very simple and the process of data acquisition and analysis can be fully automated.

One of the major obstacles in using the correlation method is that spurious results may appear, due to fluctuations in the data. The method of structure functions seems to be very effective in avoiding this problem. Also, the two algorithms to reduce the number of calculations have markedly shortened the time required for data analysis.

Acknowledgments

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