# A STEREOSCOPIC SYSTEM FOR PANORAMIC MEASUREMENT

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### **ABSTRACT**

This paper shows how a panoramic imaging system can be used to pinpoint the locations of objects surrounding it. Stereoscopic measurements are made by digitally recording two annular images obtained by translating a panoramic annular lens (PAL) along its optical axis. The images of all objects that remain within the field of view move radially as the lens is translated. The pixel shift is determined either by making direct measurements in the annular images, or, by applying a digital correlation algorithm to linearized quadrants. Since the pixel shift is related to the distance between the object and the initial position of the PAL, and the radial position angle is known from the pixel location in the image(s), the location of the object in space is uniquely determined. Initial feasibility is demonstrated and the method is applied to contour a segmented mirror designed for use in a space based telescope.

## INTRODUCTION

The invention of the panoramic annular lens (PAL) [1] led to the science of radial metrology [2]; the process of using panoramic imaging systems for inspection and measurement [3]. Prior attempts to contour regions surrounding the PAL have relied on structured light [4], or, optical techniques such as moiré [5] and speckle metrology [6]. This paper describes efforts to develop a stereoscopic imaging system for panoramic measurement.

# THE PANORAMIC ANNULAR LENS (PAL)

The stereoscopic system relies on the panoramic annular lens (PAL) shown in Fig. 1. Rays leaving points A and B are refracted upon contacting the first spherical surface, and reflect off the rear mirrored spherical surface. They travel forward in the lens and strike the front mirrored spherical surface. Reflected back, the rays are refracted at the rear flat optical surface and diverge as they exit the lens. The divergent rays leaving the flat optical surface at the back of the PAL can be "back traced" to form virtual images corresponding to points A and B at the points labeled A' and

B'. A biconvex lens, labeled as the collector lens, forms real images of these internal points at A" and B". Imaging all points contained within the field of view produces a flat annular image.

As illustrated in Fig. 2, the optical axis of the PAL is defined by a line perpendicular to the flat surface which passes through the centers of curvature of the three spherical surfaces. A longitudinal axis, labeled Z, is chosen to coincide with the optical axis. Two other axes, labeled X and Y, are established in a plane defined by the physical equator of the lens. They are chosen to form a right handed triad with the longitudinal axis. Cylindrical  $(r,\theta,z)$  coordinates may also be defined with respect to the origin in real space; the angle  $\theta$  is called the radial position angle.

The field of view can be quantified by tracing the paths of the rays that emanate from the points surrounding the lens and emerge parallel to the optical axis [7]. These rays are analogous to the chief rays in a conventional lens system and provide an accurate approximation for the limits on the field of view; since, the acceptance angles for the points surrounding the lens are small, and the collector lens restricts off-axis rays. The paraxial approximation, that assumes that points image to points as opposed to regions, leads to the conclusion that all rays in object space intersect at a common point called the entrance pupil; defined as the image of the aperture stop in object space.

Figure 3, for example, shows a 38.9 mm (1.53 in.) diameter PAL made from Shott SF14 glass having an index of refraction of 1.76. In the Y,Z plane, where the radial position angle is 90°, the position of the entrance pupil in Cartesian coordinates is (0, 1.342, 6.740) mm [(0, 0.053, 0.265) in.]. The field angle,  $\varphi$ , can be included as one of three spherical coordinates  $(\rho,\varphi,\theta)$  measured from a local system situated at the entrance pupil. For -Ti/2  $\leq \varphi \leq \text{Ti/2}$  and  $0 \leq \theta \leq 2\text{Ti}$ ,

$$x = r \cos \theta = o_p + \rho \cos \phi \cos \theta$$
  
 $y = r \sin \theta = o_p + \rho \cos \phi \sin \theta$   
 $z = h_p + \rho \sin \phi$  (1)

where the offset,  $o_p$ , and height,  $h_p$ , of the entrance pupil are equal to 1.342 mm (0.053 in.) and 6.740 mm (0.265 in.), respectively.

The angular field of view for the lens is defined by -18.81°  $\leq \varphi \leq 26.58^\circ$ . When the area between these extremes is rotated around the optical axis through a radial position angle of  $2\pi$ , a cylinder is described. This continuous region is mapped by the collector lens onto a flat annular image.

As illustrated in Fig. 3, the image space is defined by either Cartesian (x',y') or polar (r',\theta') coordinates measured from an origin situated at the center of the annulus. The X' and Y' axes in the image plane are oriented parallel to the X and Y axes in real space; and,  $\theta=\theta'$ . Points located on the inner radius, at a radial distance of  $r_i'$  in the image plane, correspond to objects viewed at  $\varphi=26.58^\circ$ ; points located on the outer radius at  $r_o$ ', correspond to objects viewed at  $\varphi=-18.81^\circ$ . In general,

$$\phi = 14.338 \ h^2 - 59.661 \ h + 26.535 \tag{2}$$

where

$$h = \frac{r_{\rho}' - r_{i}'}{r_{\rho}' - r_{i}'} \tag{3}$$

is the non-dimensional increment of the annular image measured in the image plane along r', from  $r_i'$  to the location of point P  $(r'_n)$ . The expressions are valid for  $r_i' \le r' \le r_o'$ .

Equations (2) and (3) show that the field angle to any point, P, can be determined by making measurements on the annular image. Although the radial position angle can also be measured directly there, the distance from the lens to the point is ambiguous. In Fig. 3, for example, an object located in real space anywhere along line O<sub>e</sub>P would be imaged to the same region in the annular image. The only difference would be that the size of the object would become smaller as the distance between the object and the lens increased. Thus, the only way that the location of an object surrounding the lens could be determined would be to know its size. An alternative approach is to compare the positions of the object in two annular images recorded before and after the PAL is translated along its optical axis. This approach is described below.

### STEREOSCOPIC ANALYSIS OF PANORAMIC IMAGES

Figure 4 shows a schematic of the r,z plane for a point P located in real space at a fixed radial position angle,  $\theta.$  The origin of the global coordinate system is chosen at the center of the PAL initially located at point  $O_1.$  A local spherical cylindrical coordinate system is defined at the entrance pupil of the lens at point  $O_{1e},\,\varphi_{p1}$  is the field angle measured to the point in question. When the lens moves down through a distance,  $\Delta,$  to point  $O_2,$  point P is observed from  $O_{2e}$  at angle,  $\varphi_{p2}.$  The distance  $O_{1e}P$  corresponds to the position vector in the spherical coordinate system, and from trigonometry,

$$\rho = \frac{\Delta \cos \phi_{p2}}{\sin (\phi_{p2} - \phi_{p1})} . \tag{4}$$

Since the radial position angle is the same for both PAL recordings, the position of P moves along the associated radial line in the image from  $r'_{p1}$  to  $r'_{p2}$ . The Cartesian coordinates of the point are found by substituting Eq. (4) into Eq. (1) with  $\phi = \phi_{p1}$  as,

$$x = o_{p} + \frac{\Delta \cos \phi_{p2}}{\sin (\phi_{p2} - \phi_{p1})} \cos \phi_{p1} \cos \theta$$

$$y = o_{p} + \frac{\Delta \cos \phi_{p2}}{\sin (\phi_{p2} - \phi_{p1})} \cos \phi_{p1} \sin \theta$$

$$z = h_{p} + \frac{\Delta \cos \phi_{p2}}{\sin (\phi_{p2} - \phi_{p1})} \sin \phi_{p1}$$
(5)

where  $o_p = 1.342 \text{ mm} (0.053 \text{ in.})$  and  $h_p = 6.740 \text{ mm} (0.265 \text{ in.})$ .

Thus, for a given  $\Delta$ , the location of the object can be found by measuring the radial position angle,  $\theta$ , and the image distances  $r'_{p1}$  and  $r'_{p2}$ . Equations (2) and (3) are used to convert these distances to  $\varphi_{p1}$  and  $\varphi_{p2}$ , respectively; and, Eq. (5) is applied.

### COMPUTATIONALLY ASSISTED MEASUREMENTS

One approach that may be applied to computationally determine the required distances in the image plane is to use the method of digital image correlation [8]. There are some problems, however, in applying the correlation algorithm directly to the annular images [6]. It is difficult to search along a radial direction, and the size of the object may change significantly when it is viewed from the lens at a different field angle. These problem can be eased by linearizing the annular images prior to correlation.

## STEREOSCOPIC ANALYSIS OF LINEARIZED IMAGES

Image linearization is accomplished by breaking the annular image into distinct quadrants. Linearizing a quadrant involves "rolling" the annular image along its outer circumference, and moving all the pixels between the contact points and the center of the image to vertical lines via a mapping function [9]. The quadrant transforms into a rectangle in which circumferential and radial lines appear horizontal and vertical, respectively. Every column in the linearized image corresponds to a different radial position angle,  $\theta_{\rm o}$ , while each pixel location in the column corresponds to a different propagation angle,  $\varphi_{\rm o}$ .

A relatively straightforward calibration procedure may be used to quantify these relationships. Once these calibrations have been performed, the process of correlating the two linearized stereoscopic images amounts to selecting a correlation window centered over the desired location in the first image; the row and column number of the point correspond to  $\varphi_{\text{p1}}$  and  $\theta_{\text{r}}$  respectively. When the lens is

displaced downward through a distance  $\Delta$  in real space, the field angle increases. Hence, the center of the window must be moved up along the column in the second image until the best match is established. The row number of this position corresponds to  $\varphi_{p2}$ . After the calibration functions are used to determine the values of  $\theta$ ,  $\varphi_{p1}$ , and  $\varphi_{p2}$ , these angles are substituted along with the known value of  $\Delta$  into Eq. (5) to pinpoint the Cartesian coordinates of the point in real space.

### STEREOSCOPIC MEASUREMENTS

The equations were verified by mounting a PAL imaging system on a CCD camera. The lens was oriented with its optical axis in the vertical direction and the camera was mounted on a kinematic stage that allowed it to be moved downward. Targets were situated at different field angles and at varying distances from the lens.

The images required for stereoscopic measurement were stored as 512 x 512 pixel arrays, having 256 intensity levels, using a Matrox board and IPPLUS software. When annular and linearized images were analyzed to obtain the coordinates of the targets, the results agreed to within a few percent of the locations obtained by physical measurement.

## APPLICATION TO SEGMENTED MIRRORS

The optics and the astronomy communities acknowledge that segmented mirror systems represent the future of large scale optics and interest in this area is rapidly growing. As compared to a monolithic mirror, segmented systems can be assembled from smaller mirrors that can be more easily and cheaply manufactured, often to greater tolerances. The benefits associated with such systems are apparent in situations where size poses a constraint. In the case of a telescope designed to function in space, for example, the size of the launch vehicle limits the diameter of the mirrors from which the optics can be constructed.

A major problem associated with segmented systems is the alignment and control of the individual elements. The stereoscopic PAL system offers great potential in this regard. Figure 5, for example, shows a photograph taken of the stereoscopic PAL system installed in a test bed located at the Phased Array Mirror Extendible Large Aperture (PAMELA) facility at NASA's Marshall Space Flight Center. The primary mirror assembly in this telescope consists of a 1.523 m (5 ft) diameter, spherical segmented mirror composed of 18 hexagonal segments each measuring 7 cm (2.75 in.) flat to flat. The segments are designed so that the telescope can be actively aligned while viewing objects from space.

Stereoscopic images were recorded to check the radius of curvature of the primary mirror. This was accomplished by removing the pilot tube, positioning the lens at the center of the segmented array, and then translating it along the optical axis. Figure 6 shows the annular image recorded with the lens in its initial position. The radius of the mirror obtained by analyzing such images agreed to within a few percent of the specified value.

### CONCLUSION

This paper has illustrated that it is possible to pinpoint the locations of objects surrounding a panoramic annular lens simply by moving the lens along its optical axis and digitally recording and correlating points contained in two different images. The range and resolution of the system depends upon several factors including the size, shape, and magnification of the objects; the proximity of the object to the lens; and, the displacement initiated to the lens between recordings. Although these issues have been reserved for a future paper, it should be apparent that the stereoscopic system has significant practical value with potential applications ranging from contour mapping of cavities to optical recognition systems designed to aid in robotic manipulation.

### **ACKNOWLEDGMENT**

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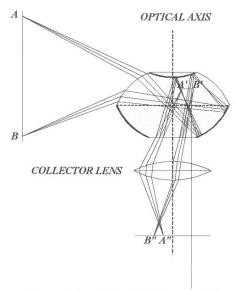


Fig 1. Panoramic annular lens (PAL).

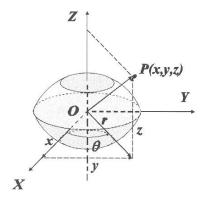


Fig. 2. Coordinates in real space.

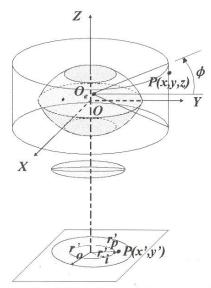


Fig. 3. The field of view and the annular image produced by a PAL.

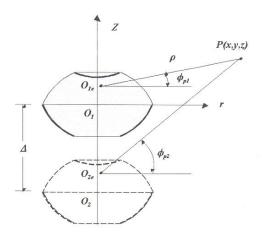


Fig. 4. Stereoscopic recording system.

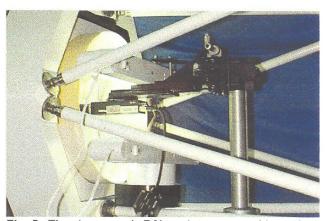


Fig. 5. The stereoscopic PAL system was used to contour a segmented mirror designed for a space based telescope.

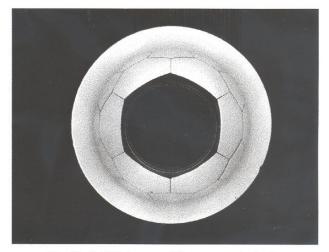


Fig. 6. A PAL image of the segmented mirror.