A Stereoscopic System for Radial Metrology

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ABSTRACT

This paper describes a technique for locating points on the surfaces of objects surrounding a panoramic imaging system. Stereoscopic measurements are made by digitally recording two annular images captured before and after a panoramic annular lens (PAL) is translated along its optical axis. The images of all objects that remain within the field of view move radially as the lens is translated. Since the pixel shift is related to the distance between the object and the initial position of the PAL, and the radial position angle of a point is known from its pixel location in the image(s), the position coordinates are uniquely determined. The study documents the analysis and calibration of the stereoscopic system and includes error analyses and feasibility tests.

INTRODUCTION

The invention of the panoramic annular lens (PAL) [1] led to the science of radial metrology; the process of using panoramic imaging systems for inspection and measurement [2]. During the inspection tasks associated with radial metrology, it is desirable to know the locations and sizes of objects contained within the field of view. Contour information is important for measurement, especially when displacement, strain, and stress must be transformed to obtain critical parameters such as principal stress or maximum shear.

Prior attempts to locate points and contour regions surrounding the PAL involved techniques such as structured lighting, moiré, and speckle metrology. But these techniques can be complex and computationally intensive, and may not always work due to geometrical constraints, occlusions, surface texture, etc. This paper presents an alternate method for making absolute and relative position measurements in the space surrounding a PAL.

THE PANORAMIC ANNULAR LENS (PAL)

As illustrated in Fig. 1, the optical axis of the PAL is defined by a line perpendicular to the rear flat surface, which passes through the centers of curvature of its three spherical surfaces. A longitudinal axis, labeled Z, is chosen to coincide with the optical axis. Two other axes, labeled X and Y, are established in a plane defined by the physical equator of the lens. They are chosen to form a right handed triad with the longitudinal axis. Cylindrical coordinates can also be defined with respect to the origin in real space; the angle \( \theta \), measured counterclockwise from the X, is called the radial position angle.

At a given \( \theta \), all rays in the object space intersect at a common point called the entrance pupil; point \( O_0 \) on Fig. 1, for example, corresponds to \( \theta = 90^\circ \). A field angle, \( \varphi \), can be included as one of three spherical coordinates \( (\rho, \varphi, \theta) \) measured from a local system situated at this point. For \(-\pi/2 \leq \varphi \leq \pi/2 \) and \( 0 \leq \theta \leq 2\pi \), the position coordinates measured relative to the Cartesian coordinate system are

\[
\begin{align*}
    x &= (o_0 + \rho \cos \varphi) \cos \theta \\
    y &= (o_0 + \rho \cos \varphi) \sin \theta \\
    z &= h_p + \rho \sin \varphi
\end{align*}
\]

where \( o_0 = 1.342 \text{ mm (0.053 in.)} \) and \( h_p = 6.740 \text{ mm (0.265 in.)} \) for the 38.9 mm (1.53 in.) diameter PAL used in the present study.

The PAL forms an internal virtual image of its surroundings by a combination of reflection and refraction; and the collector lens produces an inverted, flat annular image. Referring to Fig. 1, the image space is defined by either Cartesian \((x', y')\) or polar \((r', \theta')\) coordinates measured from an origin situated at the center of the annulus. Points located on the inner radius, at a radial distance of \( r'_c \) in the image plane, correspond to objects viewed at the maximum field angle; points located on the outer radius at \( r'_o \); correspond to objects viewed at the minimum field angle.

In practice, the camera system is mounted vertically on a tripod with the mounting surface aligned along the X axis;
the Y axis is directed away from the base. An observer looking from behind the camera in the Z direction would see the X axis to their left with Y upward. From this perspective, the radial position angle, $\theta$, is measured clockwise from the X axis. When the image is viewed on a monitor, the X' axis is directed toward the right with Y' downward. The radial position angle, $\theta'$, is measured clockwise from the X' axis.

STEREOSCOPIC ANALYSIS OF PANORAMIC IMAGES

Figure 2 shows a schematic of the $r,z$ plane of a cylindrical coordinate system with point P located in real space at a fixed radial position angle, $\theta$. The origin of the global coordinate system is chosen at the center of the PAL initially located at point $O_e$. A local spherical cylindrical coordinate system is defined at the entrance pupil of the lens at point $O_w$. The field angle measured to the point in question is $\varphi_p1$.

When the lens is moved downward through a distance, $\Delta$, to point $O_w$, point P is observed from $O_w$ at angle, $\varphi_p2$. The distance $O_wP$ corresponds to the position vector in the local spherical coordinate system, and from trigonometry,

$$
\rho = \frac{\Delta \cos \varphi_p2}{\sin (\varphi_p2 - \varphi_p1)}.
$$

(2)

Since the radial position angle is the same for both PAL recordings, the point P moves along a radial line in the image from $r_p1$ to $r_p2$. As illustrated later during calibration, each of these radial coordinates corresponds to a field angle, $\varphi$, measured in degrees and given by the relation

$$
\varphi = 0.0014 (r_p')^2 - 0.8839 r_p' + 105.73.
$$

(3)

The Cartesian coordinates of the point are found by substituting Eq. (2) into Eq. (1) with $\varphi = \varphi_p1$ as,

$$
x = [ a_p + \frac{\Delta \cos \varphi_p2}{\sin (\varphi_p2 - \varphi_p1)} \cos \varphi_p1 ] \cos \theta
$$

$$
y = [ a_p + \frac{\Delta \cos \varphi_p2}{\sin (\varphi_p2 - \varphi_p1)} \cos \varphi_p1 ] \sin \theta
$$

$$
z = h_p + \frac{\Delta \cos \varphi_p2}{\sin (\varphi_p2 - \varphi_p1)} \sin \varphi_p1
$$

(4)

where $a_p = 1.342 \text{ mm (0.053 in.)}$ and $h_p = 6.740 \text{ mm (0.265 in.)}$.

Thus, for a given $\Delta$, the location of the object can be found by first measuring the radial position angle, $\theta'$, in the image plane. Since $6' = \theta$, and Eq. (3) can be used to convert the image distances $r_p'1$ and $r_p'2$ to $\varphi_p1$ and $\varphi_p2$, respectively, Eq. (4) can be applied.

SYSTEM CALIBRATION

A stereoscopic system was built by mounting a PAL imaging system on a Pulnix CCD camera. The lens was oriented with its optical axis in the vertical direction and the camera was mounted on a tripod that allowed it to be moved downward. Images were stored as 512 x 512 pixel arrays, having 256 intensity levels, using a Matrox MVP-AT frame grabber and IPPLUS software.

A calibration was performed using "fuzzy targets" consisting of small light bulbs. The lack of structure, and differences in intensity when viewed from different locations, make the bulbs a worst-case scenario in terms of target recognition. They were placed in over two hundred locations, at a number of different field angles with radial offsets (the distance measured from the bulb perpendicular to the optical axis) ranging from 30.48 cm (1 ft) to 3.05 m (10 ft). The field angle corresponding to each source was computed using trigonometry, and image-processing software was used to determine the radial positions of the sources in the images. This data was tabulated and used to generate the calibration curve shown in Figure 3. A second order polynomial was fit through the data to quantify the parametrical relation in Eq. (3).

ERROR ANALYSIS

It is apparent from the calibration curve that there is some scatter in the data on the order of $\pm$ 1.5 degrees. The scatter leads to errors, since position measurements depend upon the field angle. This is apparent in Figure 2.

In formulating Eq. (4), it was assumed that the field angles from the original and final locations of the PAL were accurately measured as $\varphi_p1$ and $\varphi_p2$, respectively. The most severe over prediction in $\rho$ occurs at point A when $\varphi_p1$ and $\varphi_p2$ are increased and decreased, respectively, by the maximum scatter. Point B, on the other hand, represents the most severe under prediction where $\varphi_p1$ and $\varphi_p2$ are decreased and increased, respectively, by the scatter. The shaded area represents the loci of all possible predictions.

Error curves can be generated for both over- and under-predictions based on the following formula:

$$
\text{Percentage Error} = \frac{\rho_{sys} - \rho_{real}}{\rho_{real}} \times 100.
$$

(5)

In general, the errors increase for smaller shifts and larger radial offset distances. When the shift is small and the offset large, the stereoscopic system may become unstable, forcing the position vector to infinity and beyond (negative). Consequently, system design is of critical importance.

PIPE SIMULATION

The parameters that need to be taken into consideration during a system design include the tolerable errors, system shifts, and the desired ranges of field angles and offset distances. For argument sake, assume that a section of a 0.91 m (3 ft) diameter cylindrical pipe is to be studied as part of an on-line sewer inspection project. The main objective of the study is to visually identify cracks but the sizes and positions of inclusions such as roots, debris, and sludge are also of interest. The sizes of these inclusions may be
several tens of centimeters (inches).

The physical dimensions of the pipe and the inclusions place design limits on the range of radial offset distances. When coupled with the system shift, the offsets restrict the range of field angles over which measurements can be made. Consequently, a first step in the design process may be to select the region over which measurements can be made. To this end, assume that all points to be studied in the dimensional analysis are initially located in the range $0^\circ < \phi_p < \phi_{\text{min}}$ that is, they lie below a plane passing through the entrance pupil and parallel to the equatorial plane of the PAL. Even though the salient features of interest (inclusions) must lie in this area of the image, the entire field of view can still be used to identify cracks.

Since points located closest to the system move further in the image plane for a given shift, a next logical step may be to establish the minimum offset. In the example at hand, the latter was specified as 30.5 cm (12 in) by assuming that the largest inclusion would protrude 15.2 cm (6 in.) into the 0.91 m (3 ft) diameter cylindrical pipe.

Since the measurement errors decrease, as the shift becomes larger, it is now desirable to determine the largest shift possible that can be given to the system. This is accomplished by assuming that a point located at the minimum offset is initially positioned at the maximum field angle; in this case, zero degrees. The shift is continuously increased until the point in question moves past $\phi_{\text{min}}$ that is, out of the field of view.

The potential scatter observed during the calibration procedure, ± 1.5 degrees in this case, must be taken into account during this calculation. For the case at hand, the application of this procedure results in a shift, $\Delta$, equal to 14.9 cm (5.9 in.). Once this maximum shift is established, errors curves can be formulated but this must be done for the most critical condition.

Since errors increase with the offset, the designer must generate the error curves corresponding to the largest value. Assuming that all inclusions protrude into the pipe, the latter is equal to 47.72 cm (18 in.). Error plots for this distance and the maximum shift, computed earlier, are shown in Figure 4. They show that the system can measure the locations of points contained within an offset range of 30.48 cm (12 in.) $< y < 45.72$ cm (18 in.) to an accuracy of 22 percent. As mentioned previously, the points in the image plane must be initially located in the range $0^\circ < \phi < -18.8^\circ$.

A simulation was run to test this scenario. To this end, a flat test card was designed so that when it was positioned in the field of view of the stereoscopic measurement system, the offsets to the seven targets drawn on the card were contained in the range 30.48 cm (12 in.) $< y < 45.72$ cm (18 in.). In addition, the spacing between the targets was adjusted so that the offset increased by 2.54 cm (1 in.) increments.

Figure 5, for example, shows a schematic diagram of the test card as if it were placed within a 0.91 m (3 ft) diameter cylindrical pipe. For practical purposes, the upper surface of the card could represent a sludge deposit in the bottom of the pipe. The configuration depicted in the figure corresponds to the case where the camera is fed into the pipe "right side up." The axes are labeled as viewed by an observer looking into the pipe from behind the camera.

The point labeled as "1" has an offset of 30.48 cm (12 in.) and is positioned at a radial position angle, $\theta = 270$ degrees. Point "2", on the other hand, has an offset of 45.72 cm (18 in.) with $\theta = 221.8$ degrees.

The images shown in Figure 6, and the data listed in Table 1, correspond to a test conducted with a shift of 14.9 cm (5.9 in.). In this case, the card was positioned so that all targets were initially at a distance of -9.59 cm (-3.78 in.) below the equator of the lens ($\phi_p = -18.7$ degrees for Point No. 1). The errors found by comparing the experimental and theoretical magnitudes of the position vectors for all points studied fall well below the predicted error bands. The latter was attributed to the fact that the targets used in the simulation were well defined as compared to the bulbs used for calibration. When the system was re-calibrated using targets that had the same structure as those tested, the scatter reduced to 0.8 degrees; thereby, reducing the maximum predicted error to 10 percent.

The ultimate performance of the system depends upon the resolution of the digitized image. Assuming that a target must fall within a given pixel and, in the most severe case, measurements are made along the diagonal (at $\theta = 45^\circ$, 135°, 225°, 315°), the best resolution that can be achieved without using interpixel interpolation is 0.7 pixels. Noting that the highest slope on the calibration curve (0.6 degrees/pixel) occurs at the largest field angle; the smallest scatter that can be achieved over the entire field of view is ± 0.4 degrees. This corresponds to about 6 percent error.

REFERENCES
Figure 1. Nomenclature for the PAL.

Figure 2. The stereoscopic system depends upon accurate determination of field angles.

Figure 3. Calibration curve for "fuzzy targets."

Figure 4. Error curves for a scatter of ±1.5 degrees.

Figure 5. Target location for pipe simulation.

Figure 6. PAL images of the simulated targets.